April 3, 2017

Warmup:

Parametric Equation

$$x = 1 + m + n$$

$$y = 2 + 2m - n$$

$$z = 3 + 5m + 3n$$

If y-int is A

$$(0, A, 0)$$

$$A = 2 + 2m - n$$

$$0 = 1 + m + n$$

$$0 = 3 + 5m + 3n$$

$$\boxed{m = -1 - n}$$

$$0 = 3 + 5(-1 - n) + 3n$$

$$0 = 3 - 5 - 5n + 3n$$

$$0 = -2 - 2n$$

$$2 = -2n$$

$$\boxed{n = -1}$$

$$m = -1 - (-1) = \boxed{0}$$

$$A = 2 + 2(0) - (-1)$$

$$= 2 + 0 + 1$$

$$\boxed{A = 3}$$

$\therefore$ the y-int is

$(0, 3, 0)$

For $z = B$,

point is $(0, 0, B)$

$$B = 3 + 5m + 3n$$
$$0 = 1 + m + n$$
$$0 = 2 + 2m - n \quad +$$

$$0 = 3 + 3m$$
$$-3 = 3m$$
$$\boxed{m = -1}$$
$$0 = 1 + (-1) + n$$
$$\boxed{0 = n}$$

$B = 3 + 5(-1) + 3(0)$

$B = 3 - 5$

$\boxed{B = -2}$

$\therefore$ the z-intercept is

$(0, 0, -2)$

$\vec{m} = \vec{YZ} = [0, 0, -2] - [0, 3, 0]$

$= [0, -3, -2] \times -1$

$\boxed{m = [0, 3, 2]}$

$$\boxed{\vec{r}: (0,3,0) + s[0,3,2]}$$

if $s = -1$

$(0,3,0) + (-1)[0,3,2]$

$= (0,0,-2)$ = other point!!

Properties of Planes

The scalar equation of a plane

is $Ax + By + Cz + D = 0$,

with the normal vector

$\hat{n} = [A, B, C]$.

↳ perpendicular to the plane

A normal vector is found by $\vec{A} \times \vec{B}$.

Ex. 1: Develop a scalar equation of the plane with point

$P_0 = (4, -1, 3)$ and

$\hat{n} = [3, 5, -2]$

Solution:

Pick (x, y, z) as any point on the plane as point P.

Thus,

$$\overrightarrow{P_0P} \circ \hat{n} = 0$$

$$P_0P = (x - 4, y + 1, z - 3)$$

$$\overrightarrow{P_0P} \circ \hat{n}$$

$$= (x-4)(3) + (y+1)(5) + (z-3)(-2) = 0$$

$$3x - 12 + 5y + 5 - 2z + 6 = 0$$

$$\boxed{3x + 5y - 2z - 1 = 0}$$

A scalar equation of the plane given through point $P_0(x_0, y_0, z_0)$ with normal $\hat{n} = (n_x, n_y, n_z)$ is given by

$$n_x x + n_y y + n_z z + D = 0$$

Ex. 2: Find a scalar equation of the plane through point $Q(2,4,-3)$ and $\hat{n} = [3, -1, 2]$.

$$n_x x + n_y y + n_z z + D = 0$$

$$3x - y + 2z + D = 0$$

to get D, sub in $Q(2,4,-3)$

$$3(2) - (4) + 2(-3) + D = 0$$

$$6 - 4 - 6 + D = 0$$

$$\boxed{D = 4}$$

$3x - y + 2z + 4 = 0$ is the scalar equation.

Ex. 3 : Rewrite the equation

$$(x, y, z) = (2, -5, 1) + s[2, -3, 0] + t[1, 1, -1]$$

as a scalar equation of the plane.

To get $\hat{n}$, do $\vec{s} \times \vec{t}$

$$\begin{array}{cccccc} i & j & k & i & j & k \\ 2 & -3 & 0 & 2 & -3 & 0 \\ 1 & 1 & -1 & 1 & 1 & -1 \end{array}$$

$3i + 0j + 2k - 0i + 2j + 3k$

$= 3i + 2j + 5k$

$\hat{n} = [3, 2, 5]$

$$3x + 2y + 5z + D = 0$$

sub in $(2, -5, 1)$

$$3(2) + 2(-5) + 5(1) + D = 0$$

$$6 - 10 + 5 + D = 0$$

$$1 + D = 0$$

$$\boxed{D = -1}$$

$\therefore 3x + 2y + 5z - 1 = 0$

is the scalar equation of the plane

Ex.4: Find the distance

from the point $A(3,-1,1)$

to the plane

$4x-8y-z=-41$.

Solution:

$\hat{n}=[4,-8,-1]$

We also need any point

on the plane.

To find a point choose any x, y and solve for z.

choose $x = 5, y = 5$

$$4(5) - 8(5) - z = -41$$

$$20 - 40 - z = -41$$

$$-z = -21$$

$$\boxed{z = 21}$$

$\therefore$ point is $(5, 5, 21)$

The perpendicular distance will be the shortest distance.

So we will project $\overrightarrow{AX}$ onto $\hat{n}$.

$$\overrightarrow{AX} = [5-3,\ 5+1,\ 21-1]$$

$$\overrightarrow{AX} = [2,\ 6,\ 20]$$

$$\left|\operatorname{proj}_{\hat{n}} \overrightarrow{AX}\right| = \left|\frac{\overrightarrow{AX} \circ \hat{n}}{\hat{n} \circ \cancel{\hat{n}}}\right| \cancel{\hat{n}}$$

$$= \frac{|\vec{AX} \circ \vec{n}|}{|\hat{n}|}$$

$$= \frac{|(2,6,20) \circ (4,-8,-1)|}{\sqrt{4^2+(-8)^2+(-1)^2}}$$

$$= \frac{|2 \cdot 4 + 6(-8) + 20(-1)|}{\sqrt{81}}$$

$$= \frac{|8-48-20|}{9}$$

$$= \frac{60}{9} = \boxed{\frac{20}{3}}$$

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#4, 5, 6, 7, 12, 16, 18, 22