April 6, 2017

Intersection of 2 Planes

These can intersect in three ways

1. one intersection (a line)
2. no intersections (parallel)
3. ∞ intersections (same plane)

Ex. 1: Determine the line which the planes intersect

$$3x + 2y + 5z = 25$$
$$4x - 3y + z = 22 \quad \times -5$$

$$3x + 2y + 5z = 25$$
$$-20x + 15y - 5z = -110$$

$$-17x + 17y = -85 \div$$
$$-17x = -17y - 85$$

$$\boxed{x = y + 5}$$

make an equation for z
in terms of y

$$4(y+5) - 3y + z = 22$$

$$4y + 20 - 3y + z = 22$$

$$z = -4y - 20 + 3y + 22$$

$$\boxed{z = -y + 2}$$

set $y = t$, giving a parametric equation

$$x = t + 5$$

$$y = t$$

$$z = -t + 2$$

$$L_1: (5,0,2) + t[1,1,-1]$$

Ex.2 : Determine the line through which the planes intersect

$$8x - 4y + z = 12$$
$$x + y - 5z = -6 \quad \times 4$$

$$8x - 4y + z = 12$$
$$4x + 4y - 20z = -24$$
$$12x - 19z = -12$$

$$12x = 19z - 12$$

$$\boxed{x = \frac{19}{12}z - 1}$$

$$\left(\frac{19}{12}z-1\right)+y-5z=-6$$

$$y-\frac{41}{12}z-1=-6$$

$$\boxed{y=\frac{41}{12}z-5}$$

to make parametric, set

$$z=t$$

$$x=\frac{19}{12}t-1$$

$$y=\frac{41}{12}t-5$$

$$z=t$$

HW p. 480

#5, 9

Please read
p. 485

Intersection of 3 Planes

Ex. 1: Find the POI

$$x - 2y + 4z = 4$$
$$2x + 4y - z = 9$$
$$3x + 4y + 2z = 8$$

$$\boxed{x = 2y - 4z + 4}$$

$$2(2y - 4z + 4) + 4y - z = 9$$

$$4y - 8z + 8 + 4y - z = 9$$

$$8y - 9z + 8 = 9$$

$$8y = 9z + 1$$

$$\boxed{y = \frac{9}{8}z + \frac{1}{8}}$$

sub into $x = 2y - 4z + 4$

$$x = 2\left(\frac{9}{8}z + \frac{1}{8}\right) - 4z + 4$$

$$x = \frac{9}{4}z + \frac{1}{4} - 4z + 4$$

$$\boxed{x = -\frac{7}{4}z + \frac{17}{4}}$$

Now sub both into an equation to solve for z

$$x - 2y + 4z = 4$$

$$-\frac{7}{4}z + \frac{17}{4} - 2\left(\frac{9}{8}z + \frac{1}{8}\right) + 4z = 4$$

$$-\frac{7}{4}z + \frac{17}{4} - \frac{9}{4}z - \frac{1}{4} + 4z = 4$$

$$-7z + 17 - 9z - 1 + 16z = 16$$

$16 = 16$

$\therefore$ no solution

A matrix is a rectangular array of numbers made to solve linear systems.

An augmented matrix uses the coefficients to solve intersections of planes.

$$\left[\begin{array}{ccc:c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$x = 2, \; y = 3, \; z = 4$

$$\left[\begin{array}{ccc:c} 1 & 3 & 4 & -2 \end{array}\right]$$

$x + 3y + 4z = -2$

Row Operations (Rules)

1. Any row can be multiplied by a non-zero constant
2. Any row can be replaced by the sum of that row and another row
3. rows can be interchanged

Ex. 2: Find the intersection of planes using matrix elimination

$$x + y + 2z = -2$$
$$3x - y + 14z = 6$$
$$x + 2y = -5$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 3 & -1 & 14 & 6 \\ 1 & 2 & 0 & -5 \end{array}\right] - 1$$

$1-1 \quad 2-1 \quad 0-2 \quad -5-(-2)$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 3 & -1 & 14 & 6 \\ 0 & 1 & -2 & -3 \end{array}\right] -3 \times ①$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 0 & -4 & 8 & 12 \\ 0 & 1 & -2 & -3 \end{array}\right] \div -4$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 0 & 1 & -2 & -3 \\ 0 & 1 & -2 & -3 \end{array}\right] -②$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 0 & \boxed{1} & -2 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$