

Feb. 10, 2017

Warmup: Find inst.  
rate of change at  $x=2$   
for  $f(x) = -2x^3 + 5x^2$ .

$$m = \frac{f(a+h) - f(a)}{h}$$
$$= \frac{-2(a+h)^3 + 5(a+h)^2 + 2a^3 - 5a^2}{h}$$

$$\begin{array}{cccc}
 & & & 1 \\
 & & 1 & \\
 & 1 & & \\
 & & 2 & \\
 & 1 & 1 & \\
 1 & 3 & 3 & 1 \\
 a^3 & + 3a^2h & + 3ah^2 & + h^3
 \end{array}
 \leftarrow (a+h)^2$$

$$= \frac{-2(a^3 + 3a^2h + 3ah^2 + h^3) + 5(a^2}{h}$$

$$= \frac{-\cancel{2}a^3 - 6a^2h - 6ah^2 - 2h^3 + \cancel{5}a^2}{h}$$

$$= \frac{-6a^2h - 6ah^2 - 2h^3 + 10ah}{h}$$

$$\boxed{m = -6a^2 - 6ah - 2h^2 + 10a}$$

$$a = 2, h = 0$$

$$m = -6(2)^2 - 0 - 0 + 10(2) + 0$$

$$m = -4$$



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$$+ 2ah + h^2) + 2a^3 - 5a^2$$


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$$+ 10ah + 5h^2 + \cancel{2a^3} - \cancel{5a^2}$$


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$$+ 5h^2$$


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$$+ 5h$$

p. 21  
#14a

$$s(t) = -4.9t^2 + 15t + 1$$

a) find avg. rate of change

$$1 \leq t \leq 1+h$$

$$a = 1$$

$$h = h$$

$$m = \frac{f(a+h) - f(a)}{h}$$

$$= \frac{-4.9(a+h)^2 + 15(a+h) + 1 + 4.9a^2}{h}$$

Warmup 2:

$$\frac{2x^2 - 2x - 19}{x - 2} < 5$$

$$\frac{2x^2 - 2x - 19}{x - 2} - 5 < 0$$

$$\frac{-15a - 1}{}$$

$$\frac{2x^2 - 2x - 19 - 5(x-2)}{x-2} < 0$$

$$\frac{2x^2 - 2x - 19 - 5x + 10}{x-2} < 0$$

$$\frac{2x^2 - 7x - 9}{x-2} < 0$$

$$\frac{2x^2 + 2x - 9x - 9}{x-2} < 0$$

$$\frac{2x(x+1) - 9(x+1)}{x-2} < 0$$

$$\frac{(x+1)(2x-9)}{(x-2)} < 0$$

$$(-\infty, -1) \mid (-1, 2) \mid (2, 9/2) \mid (9/2, \infty)$$

$x+1$	-	+	+	+
$2x-9$	-	-	-	+
$x-2$	-	-	+	+
Overall	neg	pos	neg	pos
Test	-5	0	3	100

$$(-\infty, -1), (2, 9/2)$$

## Tangent Word Problems

$$\text{Avg. velocity} = \frac{\Delta d}{\Delta t}$$

$$\text{Inst. velocity} \quad v = \frac{d(a+h) - d(a)}{h}$$

where  $h \rightarrow 0$



Ex. 1 · A toy rocket is launched straight up so that its height,  $d$ , in metres at time  $t$  in seconds is given by

$$d(t) = -5t^2 + 30t + 2.$$

What is its velocity at  $t = 4$ ?

$$v = \frac{d(a+h) - d(a)}{h}$$

$$= \frac{-5(a+h)^2 + 30(a+h) + 2 + 5a^2 - 30a}{h}$$

$$= \frac{-5(a+h)(a+h) + 30a + 30h + 2}{h}$$

$$= \frac{-\cancel{5}a^2 - 10ah - 5h^2 + \cancel{30}a + 30h}{h}$$

$$= \frac{-10ah - 5h^2 + 30h}{h}$$

$$v = -10a - 5h + 30$$

$$v(4) = -10(4) + 30$$

$$v(4) = -10$$

$\therefore$  the speed is  
 $\cdot 10 \text{ m/s (down)}$ .

$$\underline{+5a^2 - 30a - 2}$$

$$\underline{+\cancel{2} + \cancel{5}a^2 - \cancel{30}a - \cancel{2}}$$

Ex. 2 : If  $f(x) = \sqrt{3x-2}$ ,  
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 Find inst. rate of change at  
 $x = 3$ .

$$m = \frac{f(a+h) - f(a)}{h}$$

$$= \frac{\sqrt{3a+3h-2} - \sqrt{3a-2}}{h} \cdot \frac{\sqrt{3a+3h-2} + \sqrt{3a-2}}{\sqrt{3a+3h-2} + \sqrt{3a-2}}$$

$$= \frac{3a+3h-2 + (\cancel{\sqrt{3a+3h-2}})(\cancel{\sqrt{3a+3h-2}}) - (\cancel{\sqrt{3a-2}})(\cancel{\sqrt{3a-2}})}{h(\sqrt{3a+3h-2} + \sqrt{3a-2})}$$

$$= \frac{3\cancel{a} + 3h - \cancel{2} - 3\cancel{a} + \cancel{2}}{h(\sqrt{3a+3h-2} + \sqrt{3a-2})}$$

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$$\frac{3}{\sqrt{7} + \sqrt{7}}$$

$$\frac{\sqrt{3h-2} + \sqrt{3a-2}}{\sqrt{3h-2} + \sqrt{3a-2}}$$

$$\frac{\sqrt{3a-2} - (\sqrt{3a-2})(\sqrt{3a+3h-2}) - (3a-2)}{\sqrt{3a-2}}$$

$$= \frac{3h}{h(\sqrt{3a+3h-2}) + \sqrt{3a-2}}$$

$$= \frac{3}{\sqrt{3a+3h-2} + \sqrt{3a-2}}$$

$$a = 3, \quad h = 0$$

$$m = \frac{3}{\sqrt{3(3)+0-2} + \sqrt{3(3)-2}}$$

$$m = \frac{3}{2\sqrt{7}}$$

HW p. 20

# 17, 19, 20abc, 23, 25

Ex. 3 · Find the inst. rate of change of  $f(x) = \frac{3}{x+1}$  at

$$x = 3.$$

$$m = \frac{f(a+h) - f(a)}{h}$$

$$= \left( \frac{3}{a+h+1} - \frac{3}{a+1} \right) \div h$$

$$= \left( \frac{3(a+1)}{(a+1)(a+h+1)} - \frac{3(a+h+1)}{(a+1)(a+h+1)} \right) \div h$$

$$= \frac{3a + 3 - 3a - 3h - 3}{(a+1)(a+h+1)} \div h$$

$$= \frac{-3h}{(a+1)(a+h+1)} \div h$$

$$= \frac{-3}{(a+1)(a+h+1)}$$

$$\text{at } a = 3, h = 0$$

$$= \frac{-3}{(3+1)(3+0+1)}$$

$$= \boxed{\frac{-3}{16}}$$