Feb 13, 2017

Warmup: Find the inst. rate of change of $f(x) = \dfrac{x+3}{x-1}$ at $x = 2$.

$$m = \frac{f(a+h) - f(a)}{h}$$

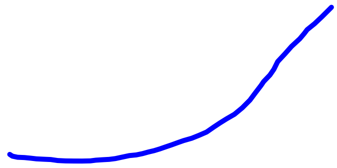
$$= \left(\frac{a+h+3}{a+h-1} - \frac{a+3}{a-1} \right) \div h$$

$$= \left[\frac{(a+h+3)(a-1)}{(a+h-1)(a-1)} - \frac{(a+3)(a+h-1)}{(a-1)(a+h-1)} \right] \div h$$

$$= \frac{\cancel{a^2} - \cancel{a} + \cancel{ah} - h + \cancel{3a} - \cancel{3} - \cancel{a^2} - \cancel{ah}}{(a+h-1)(a-1)}$$

$$= \frac{-4h}{(a+h-1)(a-1)} \div h$$

$$m = \frac{-4}{(a+h-1)(a-1)}$$



$a = 2, \; h = 0$

$$\frac{+\cancel{a} - 3\cancel{a} - 3h + \cancel{3}}{} \div h$$

$$m = \frac{-4}{(2+0-1)(2-1)}$$

$$\boxed{m = -4}$$

20a) $f(x) = 2x - x^2$

$P(1, 1)$ $Q(x, 2x - x^2)$

a, h are in terms of x

$a = 1$

$h = x - 1$

$$m = \frac{f(a+h) - f(a)}{h}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2x - x^2 - 1}{x - 1}$$

$$= \frac{-x^2 + 2x - 1}{x - 1}$$

$$= \frac{-(x^2 - 2x + 1)}{x - 1}$$

$$= \frac{-(x-1)^2}{(x-1)}$$

$$= \boxed{+1 - x}$$

23) $v(t) = 0.1(150-t)^2$

a) avg first 60

$a = 0, \ h = 60$

avg last 30

$a = 120, \ h = 30$

Limits ⊛

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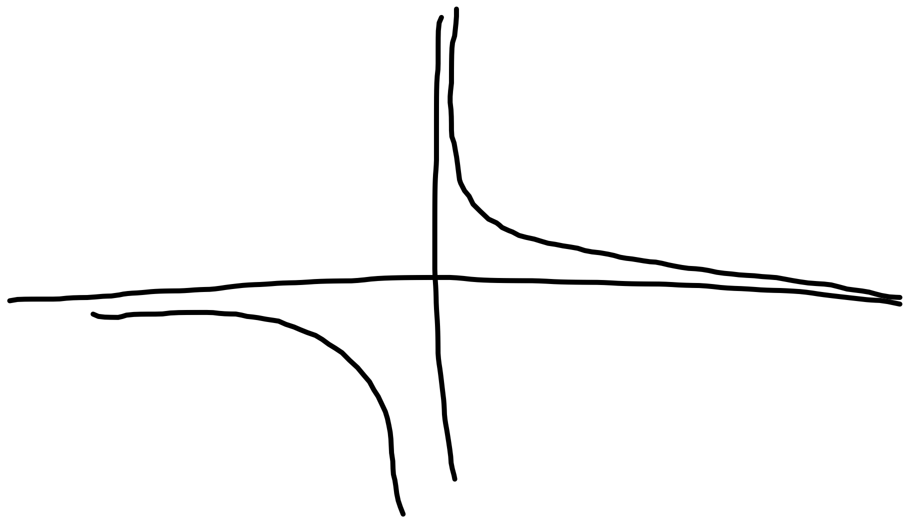
Calculus is concerned how a <u>change</u> in one quantity is related to a <u>change</u> in another. What happens to f, as x gets close to a particular value a?

Does $f(x)$ tend to "hone in" on a specific value, a limit.

A limit exists only if the values exist and are the same from both sides.

NOT A LIMIT $1/x$



Many limits can be evaluated by inspection (<u>direct substitution</u>). These are boring.

<u>Ex. 1:</u> Find

$$\lim_{x \to 2} 3x^2 - x - 4$$

$$= 3(2)^2 - (2) - 4$$

$$= \boxed{6}$$

If direct substitution ends with $\frac{0}{0}$ or $\left(\frac{\infty}{\infty}\right)$ this is called an indeterminate limit. We have the options to:

1. factor and simplify
2. rationalize
3. one sided limits
4. change of variable

Ex. 2 : Evaluate

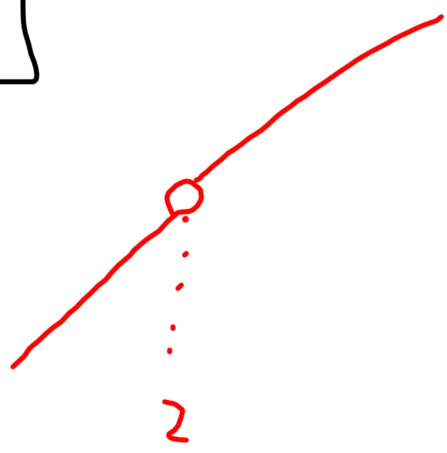
$$\lim_{x \to 2} \frac{4 - x^2}{2 - x}$$

sub $x \to 2$ $= \frac{4-4}{2-2} = \frac{0}{0}$

$$= \lim_{x \to 2} \frac{(2 \cancel{-} x)(2+x)}{2 \cancel{-} x}$$

$$= \lim_{x \to 2} 2 + x$$

$$= \boxed{4}$$



Ex. 3 : $\lim_{x \to 3} \frac{x^3 - 27}{x - 3}$

$$= \frac{3^3 - 27}{3 - 3} = \frac{0}{0}$$

$$= \lim_{x \to 3} \frac{\cancel{(x - 3)}(x^2 + 3x + 9)}{\cancel{x - 3}}$$

$$\lim_{x \to 3} x^2 + 3x + 9$$

$$= (3)^2 + 3(3) + 9$$

$$= \boxed{27}$$

Ex. 4: $\lim_{x \to 0} \frac{1 - \sqrt{1+3x}}{x}$

multiply by complex conjugate

$$\frac{1 + \sqrt{1+3x}}{1 + \sqrt{1+3x}}$$

$$\lim_{x \to 0} \frac{(1 - \sqrt{1+3x})}{x} \cdot \frac{(1 + \sqrt{1+3x})}{(1 + \sqrt{1+3x})}$$

$$\lim_{x \to 0} \frac{\cancel{1} + \cancel{\sqrt{1+3x}} - \cancel{\sqrt{1+3x}} - (\cancel{1} + 3x)}{x(1 + \sqrt{1+3x})}$$

$$= \lim_{x \to 0} \frac{-3\cancel{x}}{\cancel{x}\left(1 + \sqrt{1+3x}\right)}$$

$$= \lim_{x \to 0} \frac{-3}{1 + \sqrt{1+3x}}$$

$$= \frac{-3}{1 + \sqrt{1+0}}$$

$$= \boxed{\frac{-3}{2}}$$

Ex. 5: $\lim_{x \to 6} \dfrac{\frac{1}{x} - \frac{1}{6}}{x-6}$

$\lim_{x \to 6} \dfrac{\frac{6}{6x} - \frac{x}{6x}}{x-6}$

$\lim_{x \to 6} \dfrac{6-x}{6x} \div x-6$

$\lim_{x \to 6} \dfrac{-(\cancel{x-6})}{6x} \cdot \dfrac{1}{(\cancel{x-6})}$

$\lim_{x \to 6} \dfrac{-1}{6x} = \boxed{\dfrac{-1}{36}}$

HW #1-4 on sheet

#5-7 tomorrow