Feb. 14, 2017

a) Factor $8x^6 + 125$

sum of cubes $(2x)^3 + (5)^3$

$= (2x^2 + 5)(4x^4 - 10x^2 + 25)$

b) $\lim_{x \to 3} \dfrac{1 - \sqrt{x-2}}{x-3} = \dfrac{0}{0}$

$= \lim_{x \to 3} \dfrac{1 - \sqrt{x-2}}{x-3} \cdot \dfrac{1 + \sqrt{x-2}}{1 + \sqrt{x-2}}$

$= \lim_{x \to 3} \dfrac{1 + \cancel{\sqrt{x-2}} - \cancel{\sqrt{x-2}} - (x-2)}{(x-3)(1 + \sqrt{x-2})}$

$= \lim_{x \to 3} \dfrac{3 - x}{(x-3)(1 + \sqrt{x-2})}$

$= \lim_{x \to 3} \dfrac{-1\cancel{(x-3)}}{\cancel{(x-3)}(1 + \sqrt{x-2})}$

$$= \lim_{x \to 3} \frac{-1}{(1+\sqrt{x-2})}$$

$$= \frac{-1}{1+\sqrt{3-2}}$$

$$= \boxed{-\frac{1}{2}}$$

1d

$$\lim_{x \to 0} \frac{4x - \sqrt{x}}{3\sqrt{x}} = \frac{0}{0}$$

common factor of $\sqrt{x}$

$$\lim_{x \to 0} \frac{\cancel{\sqrt{x}}\,(4\sqrt{x} - 1)}{\cancel{\sqrt{x}}\,(3)}$$

$$= \lim_{x \to 0} \frac{4\sqrt{x} - 1}{3}$$

$$= \boxed{-\frac{1}{3}}$$

3a

$$\lim_{x \to -2} \frac{x^3+8}{x+2}$$

$$\lim_{x \to -2} \frac{\cancel{(x+2)}(x^2-2x+4)}{\cancel{x+2}}$$

$$\lim_{x \to -2} x^2-2x+4$$

$$=(-2)^2-2(-2)+4$$

$$=\boxed{12}$$

b) $\lim_{x \to 1/2} \frac{8x^3 - 1}{6x^2 - 5x + 1}$ m: 6 a: -5

$= \lim_{x \to 1/2} \frac{(2x-1)(4x^2+2x+1)}{6x^2 - 3x - 2x + 1}$

$= \lim_{x \to 1/2} \frac{(2x-1)(4x^2+2x+1)}{3x(2x-1) - 1(2x-1)}$

$= \lim_{x \to 1/2} \frac{\cancel{(2x-1)}(4x^2+2x+1)}{(3x-1)\cancel{(2x-1)}}$

$$= \lim_{x \to 1/2} \frac{4x^2+2x+1}{3x-1}$$

$$= \frac{3}{1/2}$$

$$= \boxed{6}$$

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c) $$\lim_{x \to -2} \frac{2x^4-32}{x+2}$$

$$= \lim_{x \to -2} \frac{2(x^4-16)}{x+2}$$

$$= \lim_{x \to -2} \frac{2(x^2-4)(x^2+4)}{x+2}$$

$$= \lim_{x \to -2} \frac{2(x-2)\cancel{(x+2)}(x^2+4)}{\cancel{x+2}}$$

$$= \lim_{x \to -2} 2(x-2)(x^2+4)$$

$$= \boxed{-64}$$

4c

$$\lim_{x \to 0} \frac{\frac{1}{a} - \frac{1}{x+a}}{x}$$

$$= \lim_{x \to 0} \frac{\frac{x+a}{a(x+a)} - \frac{a}{a(x+a)}}{x}$$

$$= \lim_{x \to 0} \frac{\cancel{x}}{a(x+a)} \div \cancel{x}$$

$$= \lim_{x \to 0} \frac{1}{a(x+a)}$$

$$= \boxed{\frac{1}{a^2}}$$

Substitution to Solve Limits

Ex. 1 : Evaluate

$$\lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1}$$

Sol'n: This can also be solved by rationalizing.

Set $v = \sqrt{x}$

$$\lim_{x \to 1} \frac{v - 1}{v^2 - 1}$$

$$= \lim_{x \to 1} \frac{\cancel{v-1}}{(v+1)(\cancel{v-1})}$$

$$= \lim_{x \to 1} \frac{1}{v+1}$$

but, $v = \sqrt{x}$, so

$$= \lim_{x \to 1} \frac{1}{\sqrt{x}+1}$$

$$= \boxed{\frac{1}{2}}$$

Ex.2 :

$$\lim_{x \to 9} \frac{x^{3/2} - 27}{\sqrt{x} - 3}$$

$$x^{3/2} = (\sqrt{x})^3$$

Let $c = \sqrt{x}$

$$\lim_{x \to 9} \frac{c^3 - 27}{c - 3}$$

$$= \lim_{x \to 9} \frac{\cancel{(c-3)}(c^2 + 3c + 9)}{\cancel{c-3}}$$

$$= \lim_{x \to 9} c^2 + 3c + 9$$

sub back $c = \sqrt{x}$

$$= \lim_{x \to 9} x + 3\sqrt{x} + 9$$

$$= (9) + 3\sqrt{9} + 9$$

$$= \boxed{27}$$

Ex. 3: $\lim_{x \to 0} \frac{(x+8)^{1/3} - 2}{x}$

set $b = (x+8)^{1/3}$

$b^3 = x + 8$

$\boxed{b^3 - 8 = x}$

$\lim_{x \to 0} \frac{b - 2}{b^3 - 8}$

$$\lim_{x\to 0} \frac{\cancel{b-2}}{(\cancel{b-2})(b^2+2b+4)}$$

$$= \lim_{x\to 0} \frac{1}{b^2+2b+4}$$

$$= \lim_{x\to 0} \frac{1}{(x+8)^{2/3}+2(x+8)^{1/3}+4}$$

$$= \frac{1}{4+4+4}$$

$$= \boxed{\frac{1}{12}}$$