Feb. 15, 2017

a) $\lim_{x \to 2} \dfrac{x^3 - 8}{2x - 4}$

$= \lim_{x \to 2} \dfrac{(x-2)(x^2+2x+4)}{2(x-2)}$

$= \lim_{x \to 2} \dfrac{x^2+2x+4}{2}$

$= \dfrac{4+4+4}{2}$

$= \boxed{6}$ as $x \to 2, y \to 6$

b) $\lim_{x \to 2} \dfrac{x^2 + x - 6}{\sqrt{x} - 2}$

$= \lim_{x \to 2} \dfrac{(x-2)(x+3)}{\sqrt{x-2}} \cdot \dfrac{\sqrt{x-2}}{\sqrt{x-2}}$

$= \lim_{x \to 2} \dfrac{(x-2)(x+3)\sqrt{x-2}}{(x-2)}$

$= \lim_{x \to 2} (x+3)\sqrt{x-2}$

$= (2+3)\sqrt{2-2}$

$= \boxed{0}$

5b $\lim_{x \to 0} \dfrac{x}{1+\sqrt{1+x^2}}$

$= \dfrac{0}{2} = \boxed{0}$

5c $\lim_{x \to 1} \dfrac{\sqrt{x+8}-3}{x-1}$

set $\ell = \sqrt{x+8}$

$\ell^2 = x+8$

$\boxed{\ell^2 - 8 = x}$

$$\lim_{x \to 1} \frac{t - 3}{t^2 - 8 - 1}$$

$$\lim_{x \to 1} \frac{t - 3}{t^2 - 9}$$

$$\lim_{x \to 1} \frac{t - 3}{(t - 3)(t + 3)}$$

$$\lim_{x \to 1} \frac{1}{t + 3} \qquad \text{but } t = \sqrt{x + 8}$$

$$\lim_{x \to 1} \frac{1}{\sqrt{x + 8} + 3}$$

$$= \boxed{\frac{1}{6}}$$

5d $\lim_{x \to 0} \dfrac{\sqrt{x+3} - \sqrt{3}}{x}$

$\lim_{x \to 0} \dfrac{\sqrt{x+3} - \sqrt{3}}{x} \cdot \dfrac{\sqrt{x+3} + \sqrt{3}}{\sqrt{x+3} + \sqrt{3}}$

$\lim_{x \to 0} \dfrac{x+3-3}{x(\sqrt{x+3} + \sqrt{3})}$

$\lim_{x \to 0} \dfrac{\cancel{x}}{\cancel{x}(\sqrt{x+3} + \sqrt{3})}$

$\lim_{x \to 0} \dfrac{1}{(\sqrt{x+3} + \sqrt{3})}$

$= \boxed{\dfrac{1}{2\sqrt{3}}}$

7) $f(x) = ax^2 + bx + c$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{a(x+\Delta x)^2 + b(x+\Delta x) + c}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{a(x^2 + 2x\Delta x + \Delta x^2) + bx}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{ax^2 + 2ax\Delta x + a\Delta x^2 +}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{2ax\Delta x + a\Delta x^2 + b\Delta x}{\Delta x}$$

$$-ax^2-bx-c$$

$$+b\Delta x+c-ax^2-bx-c$$

$$\cancel{bx}+b\Delta x+\cancel{c}-\cancel{ax^2}-\cancel{bx}-\cancel{c}$$

$$=\lim_{\Delta x\to 0} 2ax+a\Delta x+b$$

$$=\boxed{2ax+b}$$

Derivatives

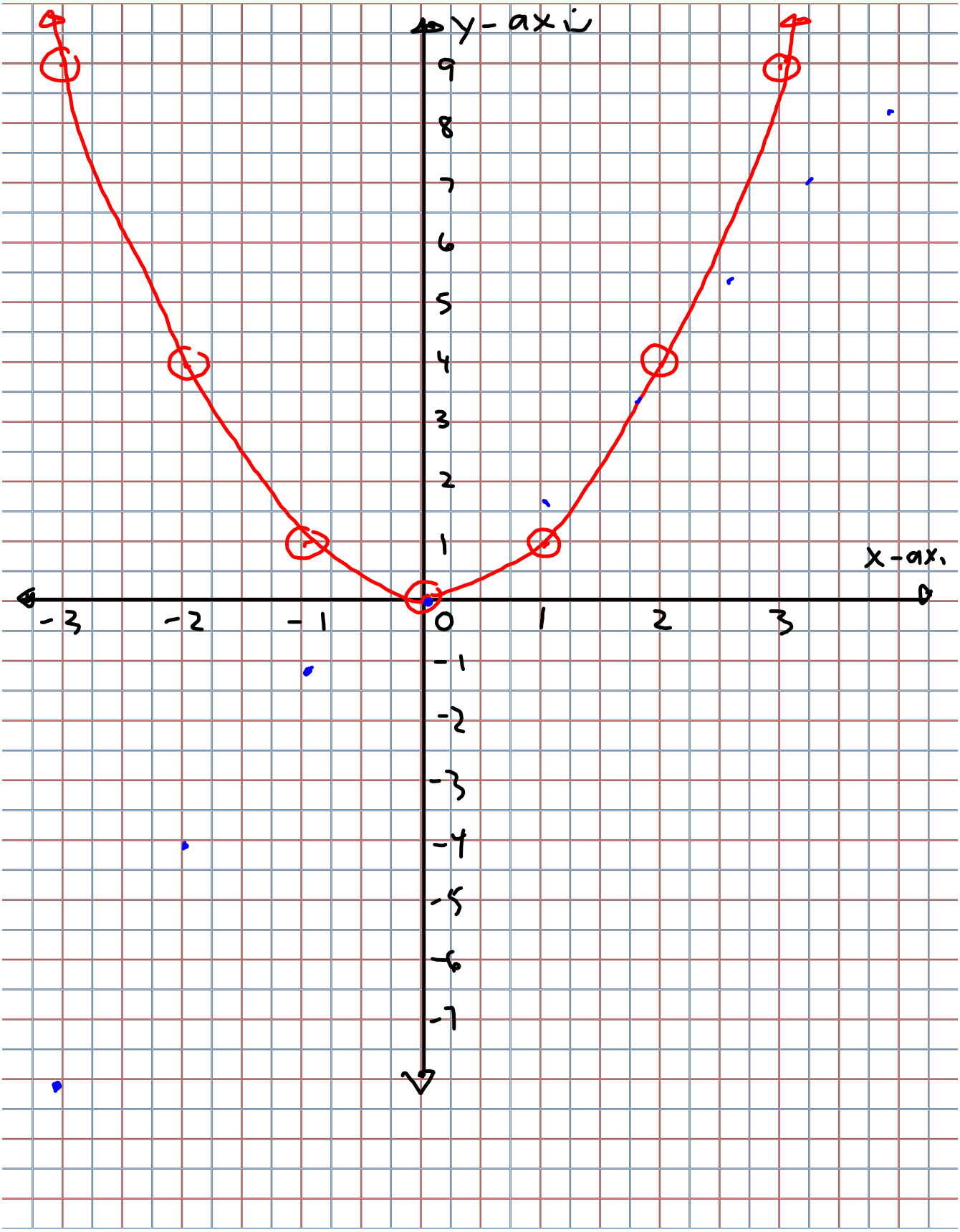
The derivative of a function is the instantaneous rate of change at any point on the graph

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

or

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$f'(x)$ or $\frac{dy}{dx}$ = "derivative of $f(x)$"


y-axis
x-axis
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-3
-2
-1
1
2
3

Ex. 1: If $f(x) = x^2$, find $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} 2x + h$$

$$\boxed{f'(x) = 2x}$$

Ex. 2: Find $f'(x)$ if

$$f(x) = -3x^3 + 2x$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{-3(x+h)^3 + 2(x+h) + 3x^3}{h}$$

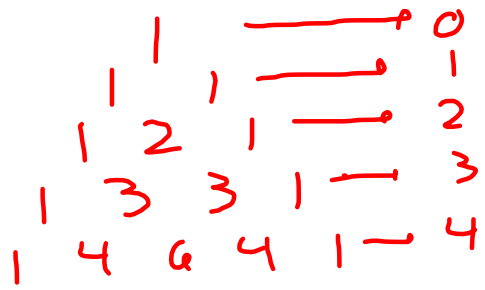
$$= \lim_{h \to 0} \frac{-3(x^3 + 3x^2h + 3xh^2}{h}$$

$$= \lim_{h \to 0} \frac{-3x^3 - 9x^2h - 9xh^2}{h}$$

$$= \lim_{h \to 0} \frac{-9x^2h - 9xh^2 - 3h^3}{h}$$

$$= \lim_{h \to 0} -9x^2 - 9xh - 3h^2 + 2$$

$$\boxed{f'(x) = -9x^2 + 2}$$



$-2x$

$+h^3) + 2x + 2h + 3x^3 - 2x$

$-3h^3 + \cancel{2x} + 2h + 3x^3 - 2x$

$+2h$

Ex.3: $f(x)=\sqrt{3x-2}$, find $f'(x)$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{3(x+h)-2}-\sqrt{3x-2}}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{3x+3h-2}-\sqrt{3x-2}}{h}\cdot\frac{\sqrt{3}}{\sqrt{}}$$

$$=\lim_{h\to 0}\frac{3x+3h-2-(3x-2)}{h\left(\sqrt{3x+3h-2}+\sqrt{3x-2}\right)}$$

$$=\lim_{h\to 0}\frac{3h}{h\left(\sqrt{3x+3h-2}+\sqrt{3x-2}\right)}$$

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$$\frac{\sqrt{x+3h-2}+\sqrt{3x-2}}{\sqrt{3x+3h-2}+\sqrt{3x-2}}$$

$$= \lim_{h\to 0} \frac{3}{\left(\sqrt{3x+3h-2}+\sqrt{3x-2}\right)}$$

$$f'(x) = \frac{3}{\sqrt{3x-2}+\sqrt{3x-2}}$$

$$\boxed{f'(x) = \frac{3}{2\sqrt{3x-2}}}$$