Feb. 16, 2017

Warmup : Find the derivative

$$f(x) = \frac{3}{x+1}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3}{x+h+1} - \frac{3}{x+1}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3(x+1)}{(x+1)(x+h+1)} - \frac{3(x+h+1)}{(x+1)(x+h+1)}}{h}$$

$$= \lim_{h \to 0} \frac{3x + 3 - 3x - 3h - 3}{(x+1)(x+h+1)} \div h$$

$$= \lim_{h \to 0} \frac{-3h}{(x+1)(x+h+1)} \div h$$

$$= \lim_{h \to 0} \frac{-3}{(x+1)(x+h+1)}$$

$$f'(x) = \frac{-3}{(x+1)^2}$$

p. 62
#28a

$$f(x) = \frac{x+2}{x-1}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x+h+2}{x+h-1} - \frac{x+2}{x-1}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(x+h+2)(x-1)}{(x+h-1)(x-1)} - \frac{(x+2)(x+h-1)}{(x-1)(x+h-1)}}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + xh + 2x - x - h - 2 - (x^2 + 2x +}{(x-1)(x+h-1)\,h}$$

$$= \lim_{h \to 0} \frac{-3h}{h(x-1)(x+h-1)}$$

$$= \lim_{h \to 0} \frac{-3}{(x-1)(x+h-1)}$$

$$f'(x) = \frac{-3}{(x-1)^2}$$

~~4x + 2h - x - 2~~)

Word Problems and Infinity Limits

Ex. 1 : A pumpkin is thrown off the top of a building that follows the path

$s(t) = -4.9t^2 + 11t + 50.$

What is the speed of the pumpkin as it hits the ground?

(1) time when height = 0.

(2) speed equation $s'(t)$

When the pumpkin hits the ground,

$$0 = -4.9t^2 + 11t + 50$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-11 \pm \sqrt{11^2 - 4(-4.9)(50)}}{2(-4.9)}$$

$\boxed{t = 4.5}$ or $\boxed{t = -2.26}$

$s'(4.5)$

$$s'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{-4.9(x+h)^2 + 11(x+h) + 50 - (}{h}$$

$$= \lim_{h \to 0} \frac{-4.9x^2 - 9.8xh - 4.9h^2 + 11x}{h}$$

$$= \lim_{h \to 0} \frac{-9.8xh - 4.9h^2 + 11h}{h}$$

$$= \lim_{h \to 0} -9.8x - 4.9h + 11$$

$$\boxed{s'(x) = -9.8x + 11}$$

$-4.9x^2 + 11x + 50)$

$+ 11h + 50 + 4.9x^2 - 11x - 50$

$s'(4.5) = -9.8(4.5) + 11$

$\boxed{s'(4.5) = -33.1}$

$\therefore$ the speed of the pumpkin as it hits the ground is -33.1.

Infinity Limits

$$\frac{3}{0^+} = \infty$$

$$\frac{3}{0^-} = -\infty$$

$$0^+ = 0.0000\ldots0001$$

$$0^- = -0.0000\ldots0001$$

$$\infty + 25 = \infty$$

$$\infty - 8000000 = \infty$$

$$300 = \infty$$

$$\frac{\infty}{25} = \infty$$

$$\frac{10}{\infty} = 0$$

Ex. 2 : $\lim_{x \to +\infty} \dfrac{3x^2 - x - 1}{2x^2 + 3x + 11} = \dfrac{\infty}{\infty}$

divide through by the highest power of x in the denominator

$$= \lim_{x \to +\infty} \frac{\frac{3x^2}{x^2} - \frac{x}{x^2} - \frac{1}{x^2}}{2\frac{x^2}{x^2} + \frac{3x}{x^2} + \frac{11}{x^2}}$$

$$= \lim_{x \to +\infty} \frac{3 - \frac{1}{x} - \frac{1}{x^2}}{2 + \frac{3}{x} + \frac{11}{x^2}}$$

$$= \boxed{\frac{3}{2}}$$

$$\lim_{x \to \infty} \frac{4x^2 - 9}{x + 1}$$

$$\lim_{x \to \infty} \frac{4x - \frac{9}{x}}{1 + 1/x}$$

$$\frac{4(\infty) - 0}{1 + 0}$$

$$= \boxed{\infty}$$