Feb. 21, 2017

Test #1 and Choice #1 start at 9:00 tomorrow

#7bc, #8 are off the assignment

#3f should have been $f(x) = 3x^3 + x^2$ but its too late

What is on the test

- definitions (explain in your own words)
- Grade 12 Review
  - long/synthetic division
  - sum/diff. of cubes

Difference Quotient

- making equations of tangents
- avg. /inst. rate of change

Limits

- factoring to cancel
- substitution
- rationalizing
- infinity

Derivatives

- word problems
- good algebra

Substitution Limit

$$\lim_{x \to 1} \frac{\sqrt[3]{x^2+3} - 2}{x-1}$$

set $a = \sqrt[3]{x^2+3}$

$$a^3 = x^2 + 3$$

$$a^3 - 3 = x^2$$

$$\sqrt{a^3 - 3} = x$$

$$\lim_{x \to 1} \frac{a-2}{\sqrt{a^3-3}-1} \cdot \frac{\sqrt{a^3-3}+1}{\sqrt{a^3-3}+1}$$

$$\lim_{x \to a} \frac{(a-2)(\sqrt{a^3-3}+1)}{a^3-2}$$

Ex. 2: $\lim_{x \to 1} \dfrac{x^2 - \sqrt{x}}{\sqrt{x} - 1}$

Set $t = \sqrt{x}$ $\quad x = t^2$

$x^2 = t^4$

$$\lim_{x \to 1} \frac{t^4 - t}{t - 1}$$

$$\lim_{x \to 1} \frac{t(t^3 - 1)}{t - 1}$$

$$\lim_{x \to 1} \frac{t\,(t-1)(t^2+t+1)}{t-1}$$

$$= \lim_{x \to 1} t\,(t^2+t+1)$$

set $t = \sqrt{x}$

$$= \lim_{x \to 1} \sqrt{x}\,(x+\sqrt{x}+1)$$

$$= \boxed{3}$$

Infinity Limits

a) $\lim_{x \to \infty} \dfrac{3x^2 - 4x + 1}{x + 1}$

$\lim_{x \to \infty} \dfrac{3x - 4 + 1/x}{1 + 1/x}$

$= \boxed{\infty}$

b) $\lim_{x \to \infty} \dfrac{4x^4 - 2x^3}{2x^4 + 1}$

$\lim_{x \to \infty} \dfrac{4 - 2/x}{2 + 1/x^4}$

$= \dfrac{4 - 0}{2 + 0} = \boxed{2}$

c) $\lim_{x \to \infty} \frac{5x}{x^2 + 1}$

$= \lim_{x \to \infty} \frac{\frac{5}{x}}{1 + 1/x^2}$

$= \frac{0}{1 + 0} = \boxed{0}$

Tangent

Find the equation of the tangent to the line $f(x) = -2x^3 + 4x$ that has slope 3.

$$m = \frac{f(a+h) - f(a)}{h}$$

$$= \frac{-2(a+h)^3 + 4(a+h) + 2a^3 - 4a}{h}$$

$$= \frac{-2a^3 - 6a^2h - 6ah^2 - 2h^3 + 4a}{h}$$

$$= \frac{-6a^2h - 6ah^2 - 2h^3 + 4h}{h}$$

$$m = -6a^2 - 6ah - 2h^2 + 4$$

$h = 0$

$$m = -6a^2 + 4$$

$+4h+2a^3-4a$

Find x-values when $m=3$

$$3=-6a^2+4$$

$$-1=-6a^2$$

$$\frac{1}{6}=a^2$$

$$a=\pm\sqrt{1/6}$$

x-values when $m=3$

$x = \sqrt{1/6}$: $x = -\sqrt{1/6}$

$y = f(\sqrt{1/6})$: $y = f(-\sqrt{1/6})$

$y = mx + b$: $y = mx + b$

$y = 3(\sqrt{1/6}) + b$: $y = 3(-\sqrt{1/6}) + b$

Avg. Rate of Change

The volume of a cylindrical pool is $V(r) = \pi r^2(20)$. What is the average rate of change from a radius of 4m to 6m?

$$m = \frac{f(a+h) - f(a)}{h}$$

$a = 4 \quad h = 2$

$$= \frac{\pi(a+h)^2 \cdot 20 - \pi a^2(20)}{h}$$

$$= \frac{20\pi(a^2+2ah+h^2) - 20\pi a^2}{h}$$

$$= \frac{40\pi ah + 20\pi h}{h}$$

$$m = 40\pi a + 20\pi h$$

$$a = 4 \quad h = 2$$

$$= 160\pi + 40\pi$$

$$\boxed{m = 200\pi}$$