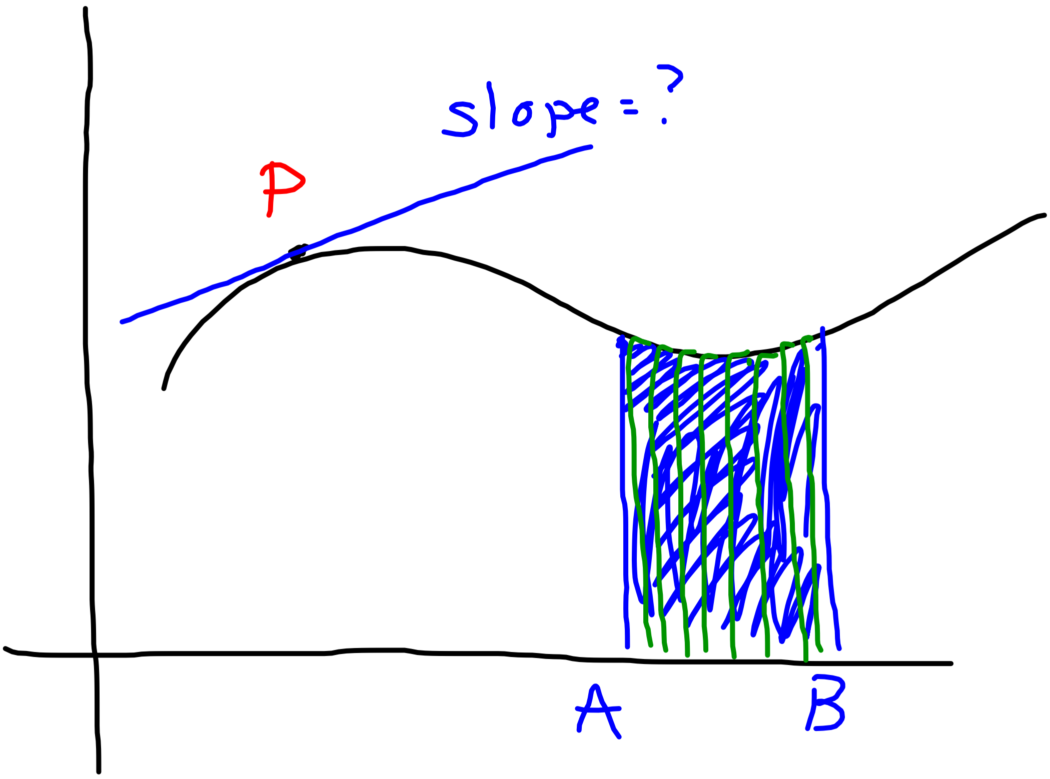
Rates of Change

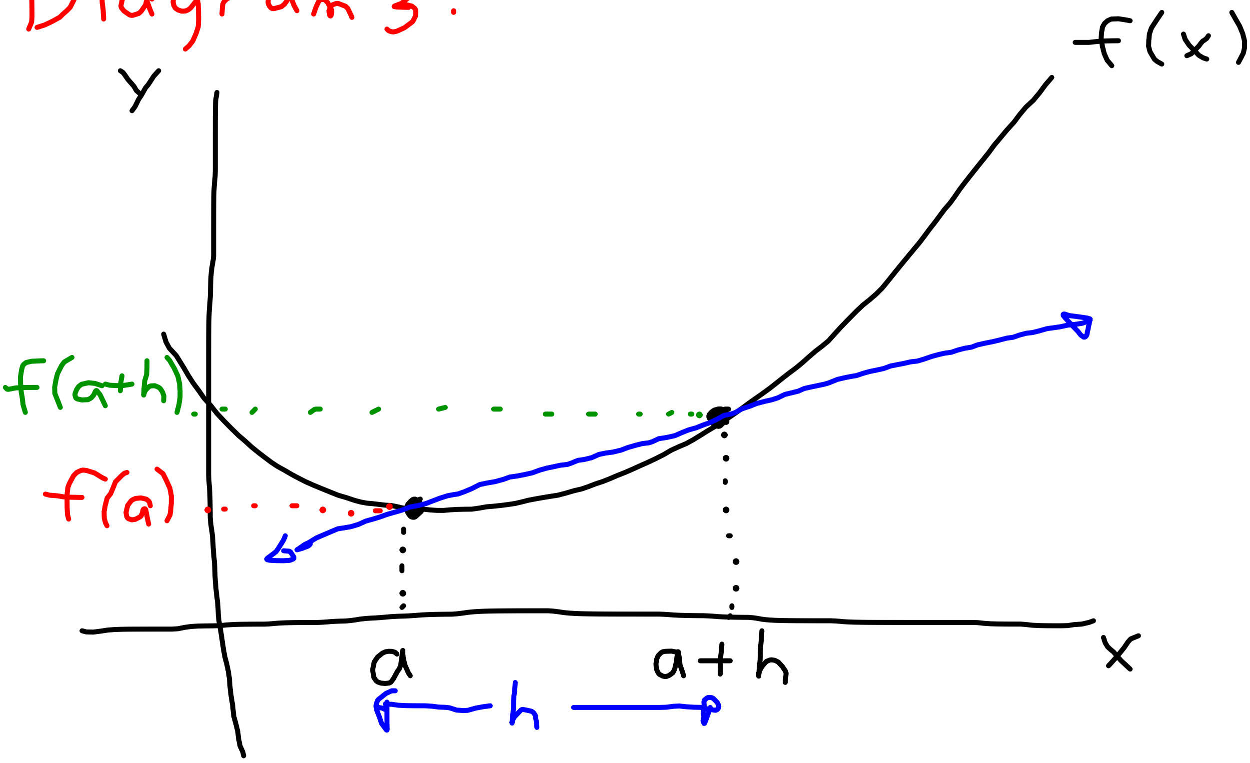
Two simple geometric problems led to the development of calculus.

1. the problem of tangents: what is the slope of the tangent to a graph at point P?
2. what is the area under a graph between points A and B


slope = ?
P
A
B

The rate of change is how rapidly the dependent variable changes when there is a change in the independent.

Diagram 3.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{f(a+h) - f(a)}{a+h-a}$$

$$\boxed{m = \frac{f(a+h) - f(a)}{h}}$$

The Difference Quotient

a = starting point

h = increment

(0 for instantaneous)

Ex. 1: Let $f(x) = -2x^2$

a) Calculate the average rate of change from $x = -2$ to $x = 3$.

$$m = \frac{f(a+h) - f(a)}{h}$$

$$= \frac{-2(a+h)^2 + 2a^2}{h}$$

$$= \frac{-2(a+h)(a+h) + 2a^2}{h}$$

$$= \frac{(-2a-2h)(a+h)+2a^2}{h}$$

$$= \frac{-\cancel{2a^2}-2ah-2ah-2h^2+\cancel{2a^2}}{h}$$

$$= \frac{-4ah-2h^2}{h}$$

$$\boxed{m = -4a-2h}$$

$$a=-2, \quad h=5$$

$$m = -4(-2)-2(5)$$

$$\boxed{m=-2}$$

b) Find the inst. rate of change at $x = 1, 3, 5,$

$x = 1 \rightarrow a = 1, \; h = 0$

$m = -4(1) - 2(0) = \boxed{-4}$

$x = 3 \rightarrow a = 3, \; h = 0$

$m = -4(3) - 2(0) = \boxed{-12}$

$x = 5 \rightarrow a = 5, \; h = 0$

$m = -4(5) - 2(0) = \boxed{-20}$

Ex. 2: If $f(x) = 3x^2 + 5x$,

a) calculate the avg. rate of change from $x = 1$ to $x = 3$

b) find the slope of the tangent at $x = 2$.

$$m = \frac{f(a+h) - f(a)}{h}$$

$$= \frac{3(a+h)^2 + 5(a+h) - 3a^2 - 5a}{h}$$

$$= \frac{\cancel{3a^2} + 6ah + 3h^2 + \cancel{5a} + 5h - \cancel{3a^2} - \cancel{5a}}{h}$$

$$= \frac{6ah + 3h^2 + 5h}{h}$$

$$\boxed{m = 6a + 3h + 5}$$

a) $a = 3, \ h = -2$

$$m = 6(3) + 3(-2) + 5$$

$$\boxed{m = 17}$$

b) $a = 2, \ h = 0$

$$m = 6(2) + 3(0) + 5$$

$$\boxed{m = 17}$$

HW p. 2 # 8, 9
p. 20 # 10, 11, 14, 15, 16