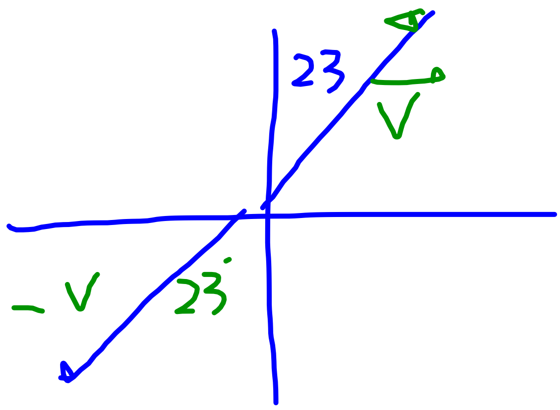
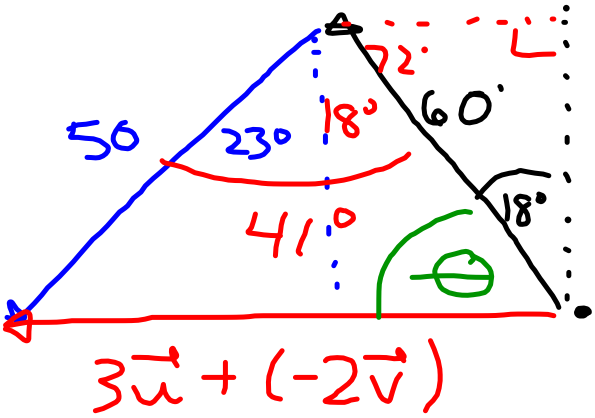
March 1, 2017

$\vec{u} = 20\text{km}[N\,18°\,W]$,
$\vec{v} = 25\text{km}[N\,23°\,E]$, find
$3\vec{u} - 2\vec{v}$.

$3\vec{u} = 60\text{km}[N\,18°\,W]$
$-2\vec{v} = 50\text{km}[S\,23°\,W]$





direction = $18° + \theta$

$$|3\vec{u} + (-2\vec{v})|^2 = 50^2 + 60^2 - 2(50)(60)\cos 41°$$

$$\boxed{|3\vec{u} - 2\vec{v}| = 39.6}$$

$$\frac{\sin\theta}{50} = \frac{\sin 41}{39.6}$$

$$\sin\theta = \frac{50\cdot\sin 41}{39.6}$$

$$\theta = \sin^{-1}\left(\frac{50\cdot\sin 41}{39.6}\right)$$

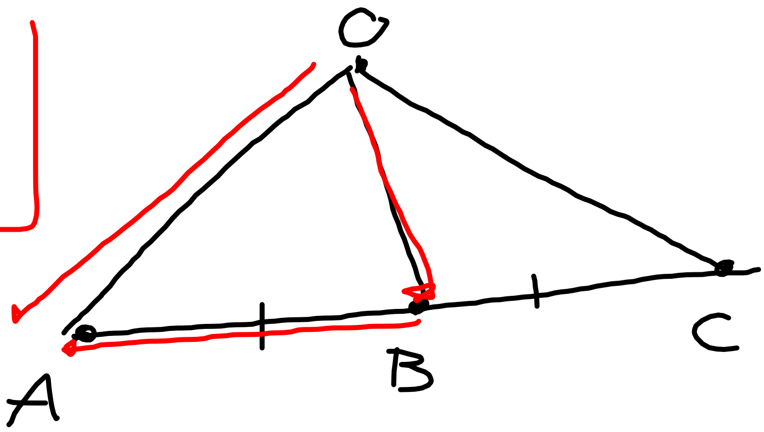
$$\boxed{\theta = 55.8^\circ}$$

$\therefore 3\vec{u} - 2\vec{v} = 39.6\,\text{km}\ [N\,73.8^\circ\,W]$

or

$= 39.6\,\text{km}\ [W\,16.2^\circ\,N]$

p.335
#17



$$\overrightarrow{AB} = \overrightarrow{BC}$$

$$\overrightarrow{OA} + \overrightarrow{OC} = 2\overrightarrow{OB}$$

$$\overrightarrow{OB} + \overrightarrow{BA} + \overrightarrow{OB} + \overrightarrow{BC}$$

$$2\overrightarrow{OB} + \overrightarrow{BA} + \overrightarrow{BC}$$

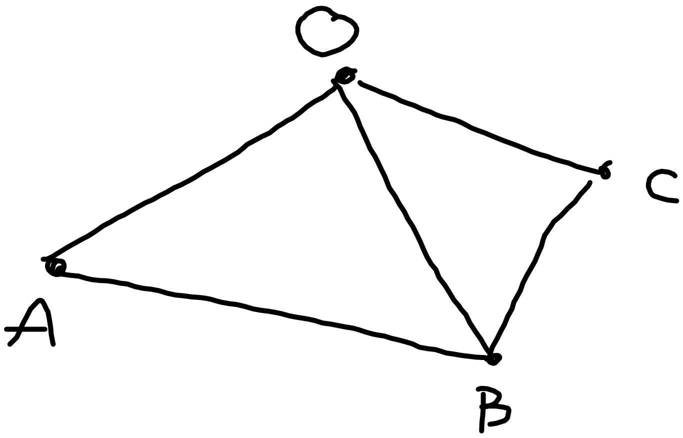
$$2\overrightarrow{OB} + (-\overrightarrow{BC}) + \overrightarrow{BC}$$

$$2\overrightarrow{OB} = 2\overrightarrow{OB}$$

20) Given $\overrightarrow{OA} + \overrightarrow{OC} = 2\overrightarrow{OB}$

prove that A, B, C are collinear

and B is midpoint



Express $\overline{OB}$ interms of $\overline{OA}$

$$\overline{OB} = \overline{OA} + \overline{AB}$$

Express $\overline{OB}$ interms of $\overline{OC}$

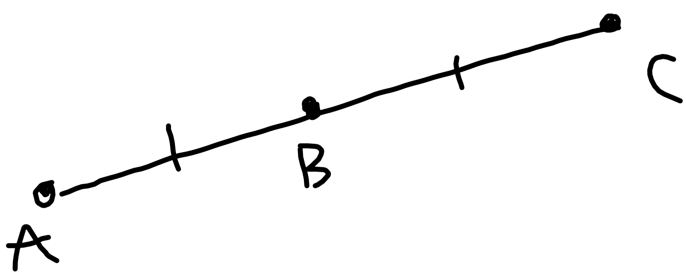
$$\overline{OB} = \overline{OC} + \overline{CB}$$

$$\therefore 2\overline{OB} = \overline{OA} + \overline{AB} + \overline{OC} + \overline{CB}$$

$$\therefore \vec{O} = \overline{AB} + \overline{CB}$$

$$-\overline{CB} = \overline{AB}$$

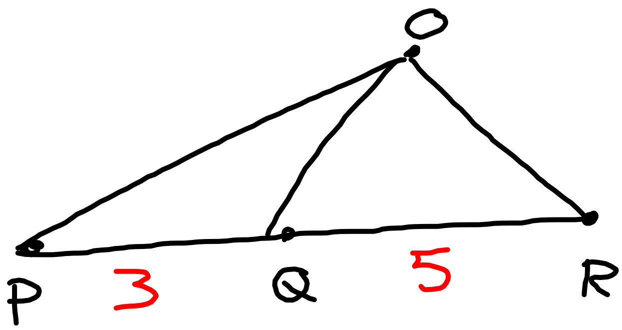
$$\overline{BC} = \overline{AB}$$



HW: Sheet
#1, 4, 6, 11

Vector Proofs

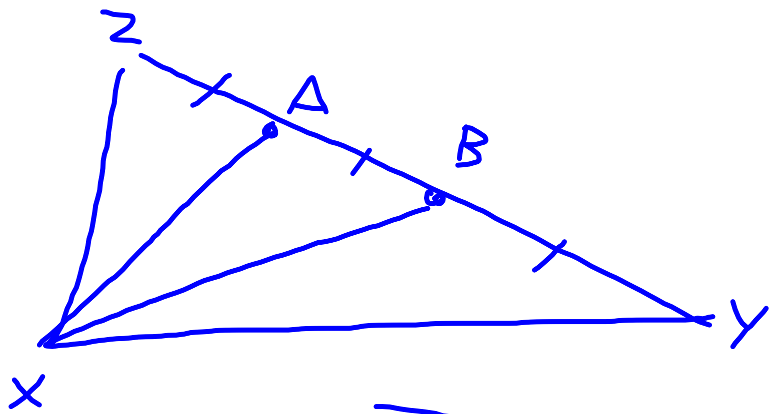
Ex. 1: Consider collinear points P, Q and R. Consider a reference point O. Write OQ as a linear combination of OP and OR if Q divides PR in the ratio 3:5.



$$
\begin{aligned}
\overline{OQ} &= \overline{OP} + \overline{PQ} \\
&= \overline{OP} + \frac{3}{8}\overline{PR} \\
&= \overrightarrow{OP} + \frac{3}{8}\left(\overrightarrow{PO} + \overrightarrow{OR}\right) \\
&= \overrightarrow{OP} + \frac{3}{8}\overrightarrow{PO} + \frac{3}{8}\overrightarrow{OR} \\
&= \overrightarrow{OP} - \frac{3}{8}\overrightarrow{OP} + \frac{3}{8}OR
\end{aligned}
$$

$$\boxed{= \frac{5}{8}\overline{OP} + \frac{3}{8}OR}$$

Ex. 2 : Prove $\overline{XA} + \overline{XB} = \overline{XY} + \overline{XZ}$



express $\overline{XA}$ in terms of $\overline{XZ}$

$$\overline{XA} = \overline{XZ} + \overline{ZA}$$

express $\overline{XB}$ in terms of $\overline{XY}$

$$\overline{XB} = \overline{XY} + \overline{YB}$$

$\therefore \sqrt{A} + \sqrt{B} =$