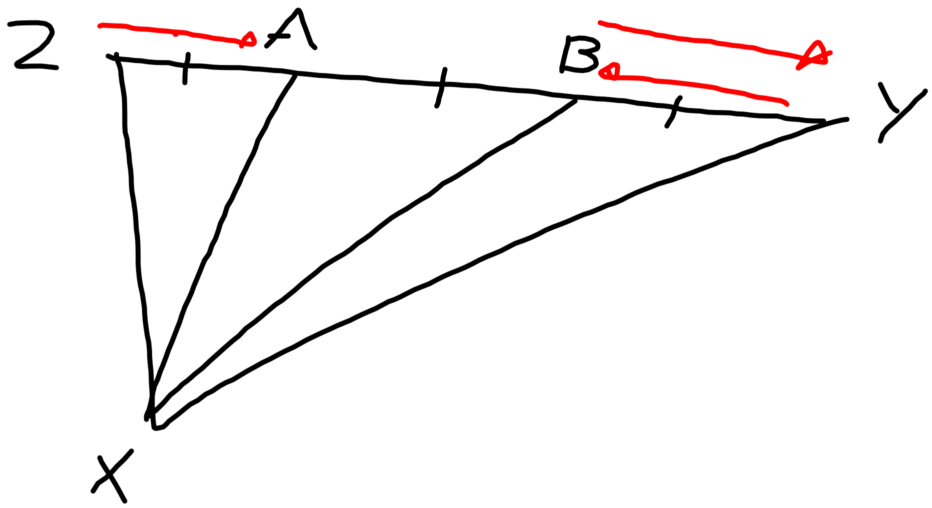
March 2, 2017



Prove $XA + XB = XY + XZ$

Express $\overline{XA}$ in terms of $\overline{XZ}$

$$\overline{XA} = \overline{XZ} + \overline{ZA}$$

$$\overline{XB} = \overline{XY} + \overline{YB}$$

$$\therefore XA + XB = \overline{XZ} + \overline{ZA} + \overline{XY} + \overline{YB}$$

$$= \overline{XY} + \overline{XZ} + \overline{ZA} + \overline{YB}$$

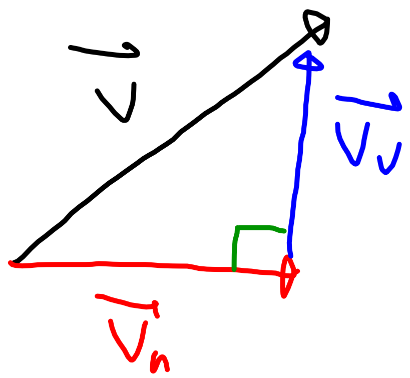
$$= \overline{XY} + \overline{XZ} + \overline{BY} + \overline{YB}$$

$$= \overline{XY} + \overline{YZ}$$

Breaking A Vector Into Components

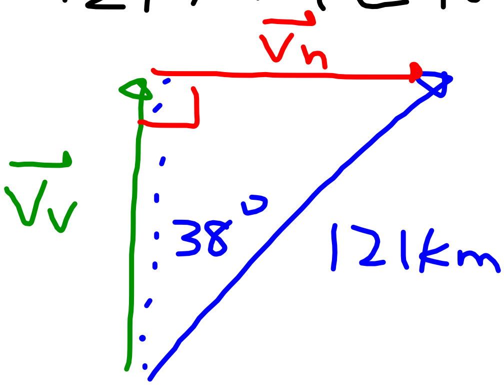
Any vector can be resolved into rectangular components.

Any vector $\vec{V}$ can be written as $\vec{V} = \vec{V_h} + \vec{V_v}$,



Both of these can be obtained using SOH CAH TOA.

Ex. 1: What are the horizontal and vertical components of $\vec{v} = 121\,km\,[N\,38°E]$.



$$\sin 38^\circ = \frac{|\vec{V_h}|}{121}$$

$$|\vec{V_h}| = 121 \cdot \sin 38^\circ$$

$$\boxed{\vec{V_h} = 74.5\text{ km [E]}}$$

$$\cos 38^\circ = \frac{|\vec{V_v}|}{121}$$

$$|\vec{V_v}| = 121 \cdot \cos 38^\circ$$

$$\boxed{V_v = 95.3\text{ km [N]}}$$

Ex. 2: Using components, if

$\vec{u} = 14 \text{ km/h [S } 30^\circ \text{ W]}$,

$\vec{v} = 21 \text{ km/h [S } 18^\circ \text{ E]}$,

what is $\vec{u} + \vec{v}$?

$\sin 30 = \frac{u_x}{14}$

$|\vec{u_x}| = 14 \cdot \sin 30°$

$\boxed{u_x = 7 km/h [W]}$

$\cos 30 = \frac{u_y}{14}$

$\overline{u_y} = 14 \cdot \cos 30°$

$\boxed{\vec{u_y} = 12.1 km/h [S]}$

$\sin 18° = \frac{V_x}{21}$

$|\overline{V_x}| = 21 \cdot \sin 18°$

$\boxed{V_x = 6.5 km/h [E]}$

$\cos 18° = \frac{V_y}{21}$

$\overline{V_y} = 21 \cdot \cos 18°$

$\boxed{\overline{V_y} = 20.0 km/h [S]}$

$$\vec{r} = \vec{u} + \vec{v}$$

$$\vec{r}_x = \vec{u}_x + \vec{v}_x$$

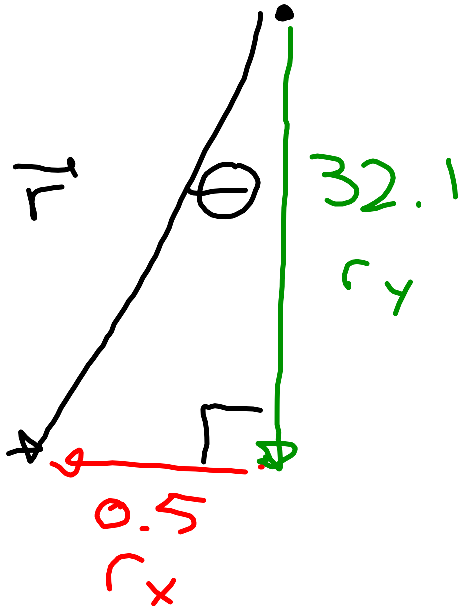
$$\vec{r}_x = 7\,km/h\,[W] + 6.5\,km/h\,[E]$$

$$\boxed{\vec{r}_x = 0.5\,km/h\,[W]}$$

$$\vec{r}_y = \vec{u}_y + \vec{v}_y$$

$$= 12.1\,km/h\,[S] + 20.0\,km/h\,[S]$$

$$\boxed{\vec{r}_y = 32.1\,km/h\,[S]}$$



$\therefore \vec{u} + \vec{v} =$
32.1 km [S 0.9° W]

$$|\vec{r}|^2 = (0.5)^2 + (32.1)^2$$

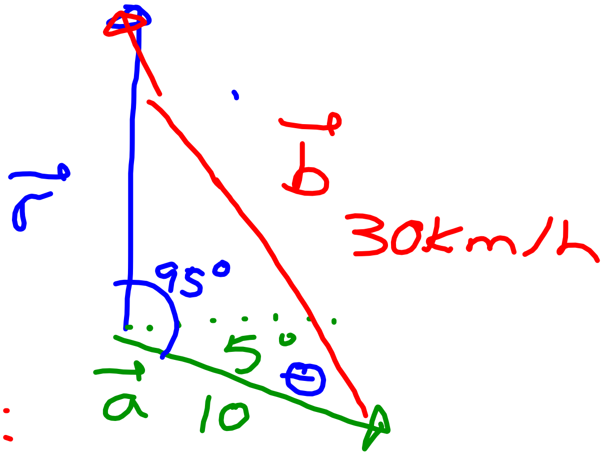
$$\boxed{|\vec{r}| = 32.1\,km}$$

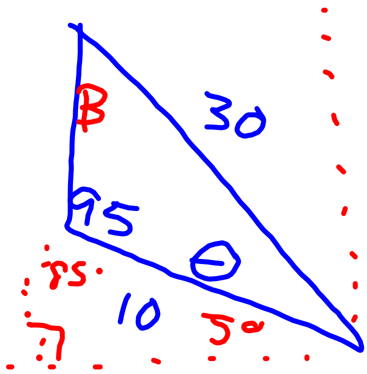
$$\tan\theta = \frac{0.5}{32.1}$$

$$\boxed{\theta = 0.9^\circ}$$

Ex.3: Bailey the snowboarder wants to head straight [N]. If there is an avalanche travelling at 10km/h [E 5°S] and Bailey travels 30km/h, at what heading must Bailey aim?

Bailey's aim + avalanche = [N]





$$\frac{\sin \beta}{10} = \frac{\sin 95}{30}$$

$$\sin \beta = \frac{10 \cdot \sin 95}{30}$$

$$\boxed{\beta = 19.4°}$$

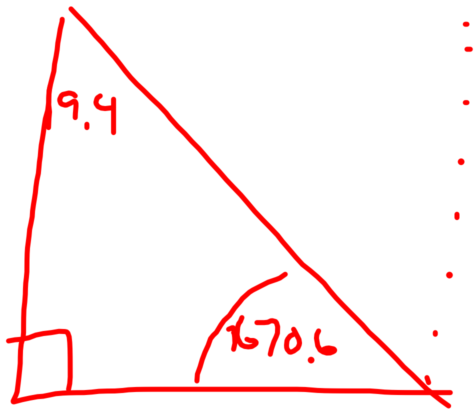
$\theta = 180° - 95 - 19.4°$

$\boxed{\theta = 65.6°} + 5°$

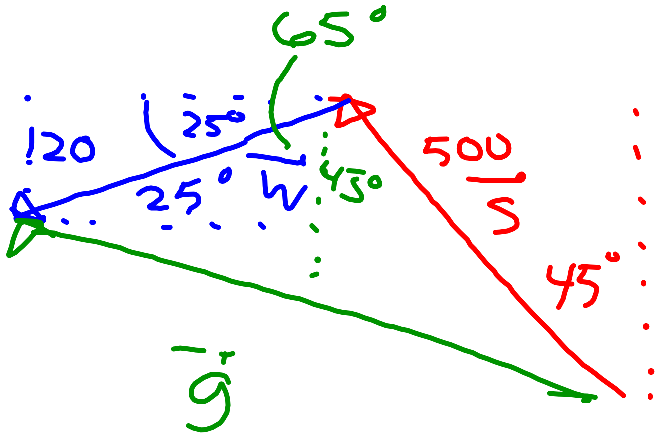
$\therefore$ Bailey must aim

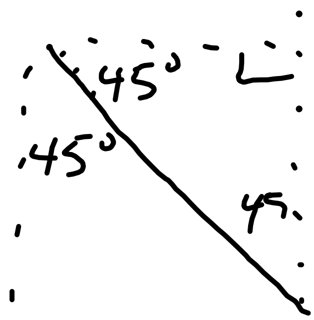
[W 70.6° N] or

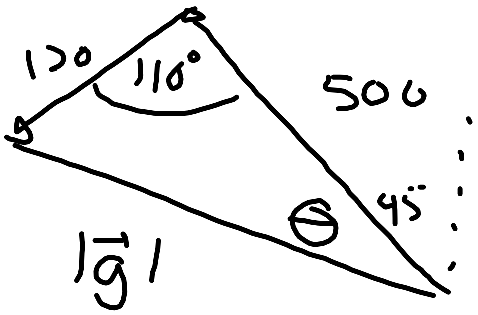
[N 19.4° W]



Ex. 4: Superman heads [NW] at 500km/h. He encounters a wind of 120km/h from 25° North of east. Find the resultant ground velocity.







heading =
$\theta + 45°$

$$|\vec{g}|^2 = 120^2 + 500^2 - 2(120)(500)\cos 110°$$

$$\boxed{|\vec{g}| = 552.66 \text{ km/h}}$$

$$\frac{\sin\theta}{120} = \frac{\sin 110°}{552.7}$$

$$\sin\theta = \frac{120 \cdot \sin 110°}{552.7}$$

$$\boxed{\theta = 11.8°}$$

∴ From the ground, Superman appears to be flying

552.66 km/h [N 56.8° W].

HW p. 343 # 6, 8, 9, 14

pg. 350 # 6, 7, 8, 9, 13, 20