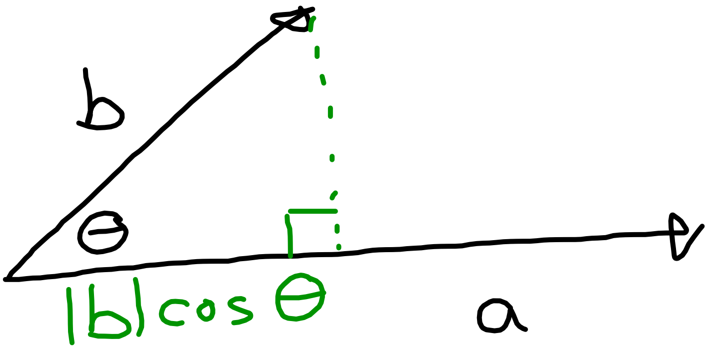
March 20, 2017

Cartesion Vector

[3, 4]

vector from (0, 0) to (3, 4)

The Dot Product



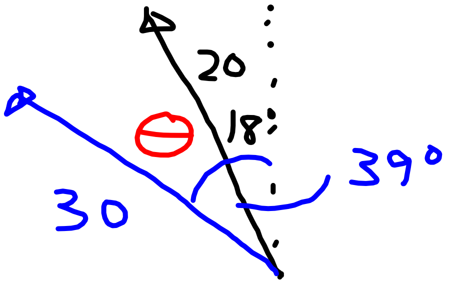
For two vectors $\vec{a}$ and $\vec{b}$, the dot product is defined as

$$\vec{a} \circ \vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

where θ is the angle between $\vec{a}$ and $\vec{b}$ when arranged tail to tail and $0^\circ \leq \theta \leq 180^\circ$.

The dot product is a scalar and represents the # of times the horizontal component of $\vec{b}$ is multiplied by $\vec{a}$.

Ex. 1: if $\vec{u} = 20\,[N\,18°\,W]$ and $\vec{v} = 30\,[N\,39°\,W]$, find $\vec{u} \circ \vec{v}$.



$\theta = 39° - 18°$

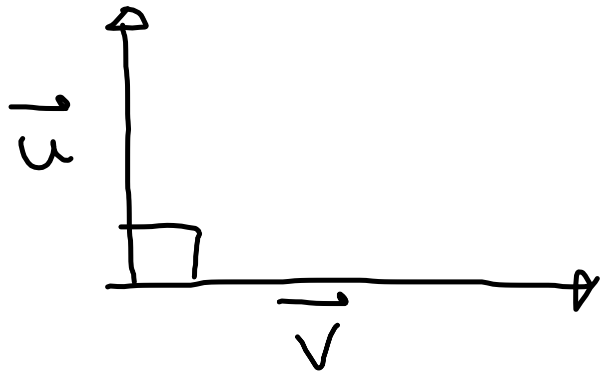
$\boxed{\theta = 21°}$

$$\vec{u} \circ \vec{v} = |\vec{u}||\vec{v}|\cos\theta$$

$$= 20 \cdot 30 \cdot \cos 21°$$

$$\boxed{\vec{u} \circ \vec{v} = 560.15}$$

Properties of The Dot Product



if $\vec{u}$ and $\vec{v}$ are perpendicular, $\vec{u} \circ \vec{v} = 0$.

$$\vec{u} \circ \vec{v} = \vec{v} \circ \vec{u}$$

$$\vec{u} \circ \vec{u} = |\vec{u}|^2$$

For any vectors $\vec{u}$, $\vec{v}$ and $k \in \mathbb{R}$,

$$\boxed{(k\vec{u}) \circ \vec{v} = k(\vec{u} \circ \vec{v}) = \vec{u} \circ (k\vec{v})}$$

For vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$,

$$\boxed{\vec{u} \circ (\vec{v} + \vec{w}) = \vec{u} \circ \vec{v} + \vec{u} \circ \vec{w}}$$

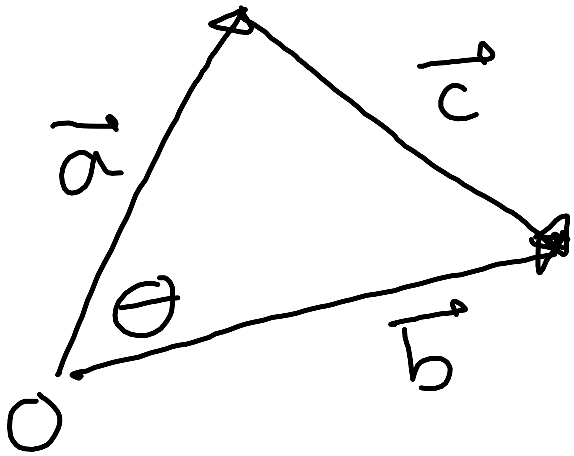
Dot Product For Cartesian Vectors

Let $\vec{a} = [a_x, a_y]$,

$\vec{b} = [b_x, b_y]$.

Then from the definition of the dot product

$$\vec{a} \circ \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$



From the diagram,

$$\vec{c} = \vec{a} - \vec{b}$$

$$= [a_x - b_x, a_y - b_y]$$

From cosine law,

$$|\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}| \cos C$$

$$2|\vec{a}||\vec{b}| \cos\theta = |a|^2 + |b|^2 - |\vec{c}|^2$$

$$2\,\vec{a} \circ \vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - |\vec{c}|^2$$

$$\vec{a} \circ \vec{b} = \frac{|\vec{a}|^2 + |\vec{b}|^2 - |\vec{c}|^2}{2}$$

$$= \frac{\left(\sqrt{a_x^2 + a_y^2}\right)^2 + \left(\sqrt{b_x^2 + b_y^2}\right)^2 - (}{2}$$

$$= \frac{a_x^2 + a_y^2 + b_x^2 + b_y^2 - [(a_x}{2}$$

$$= \frac{\cancel{a_x^2} + \cancel{a_y^2} + \cancel{b_x^2} + \cancel{b_y^2} - (\cancel{a_x^2}}{}$$

$$= \frac{2a_x b_x + 2a_y b_y}{2}$$

$$\vec{c} = [a_x - b_x, a_y - b_y]$$

$$\sqrt{(a_x - b_x)^2 + (a_y - b_y)^2})^2$$

$$- b_x)^2 + (a_y - b_y)^2]$$

$$\frac{-2a_x b_x + \cancel{b_x}^2 + \cancel{a_y}^2 - 2a_y b_y + \cancel{b_y}^2)}{2}$$

$$\boxed{\therefore \vec{a} \cdot \vec{b} = a_x b_x + a_y b_y}$$

Ex. 2: Find $\vec{u} \circ \vec{v}$

$\vec{u} = [3, 4]$, $\vec{v} = [-1, 6]$

$$\vec{u} \circ \vec{v} = (3)(-1) + 4(6)$$

$$= -3 + 24$$

$$\boxed{\vec{u} \circ \vec{v} = 21}$$

Ex. 3: Find k so that $\vec{u}$ and $\vec{v}$ are perpendicular

$\vec{u} = [4, -2]$, $\vec{v} = [k, 6]$

$$\vec{u} \cdot \vec{v} = 0$$
$$4k + (-2)(6) = 0$$
$$4k - 12 = 0$$
$$4k = 12$$
$$\boxed{k = 3}$$

HW Pg. 376

#1, 2, 3, 10, 11, 14, 17, 18