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Mar 21, 2017

3d) $(\vec{u}+\vec{v})\circ(\vec{w}+\vec{x})$

$= \vec{u}\circ\vec{w} + \vec{u}\circ\vec{x} + \vec{v}\circ\vec{w} + \vec{v}\circ\vec{x}$

$\vec{u}\circ\vec{u} = |\vec{u}|^2$

14a $\vec{a}$ and $\vec{b}$ are non-zero,

$\vec{a} \cdot \vec{b} = 0$ if $\vec{a}$ is perp to $\vec{b}$.

$\vec{a} = [a_x, a_y]$ $\vec{b} = [b_x, b_y]$

$a_x b_x + a_y b_y = 0$

$a_x b_x = -a_y b_y$

if $\vec{a}$ and $\vec{b}$ are perp, slopes are negative reciprocals.

$\frac{a_y}{a_x}$ = a's slope

$-\frac{b_x}{b_y}$ = b's slope perp. to a

$$\frac{a_y}{a_x} = -\frac{b_x}{b_y}$$

$$a_y b_y = -a_x b_x$$

$$\boxed{-a_y b_y = a_x b_x}$$

b) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

$\vec{a} = [a_x, a_y] \quad \vec{b} = [b_x, b_y]$

$a_x b_x + a_y b_y \quad \vdots \quad b_x a_x + b_y a_y$

$a_x b_x + a_y b_y = a_x b_x + a_y b_y$

Applications of Dot Product

① Angle Between Two Vectors

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

Ex. 1: Find the angle between $\vec{u} = [1, 2]$ and $\vec{v}[-3, 4]$.

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$$

$$= \frac{(1)(-3) + 2(4)}{\left(\sqrt{1^2+2^2}\right)\left(\sqrt{(-3)^2+4^2}\right)}$$

$$= \frac{-3+8}{\sqrt{5}\sqrt{25}}$$

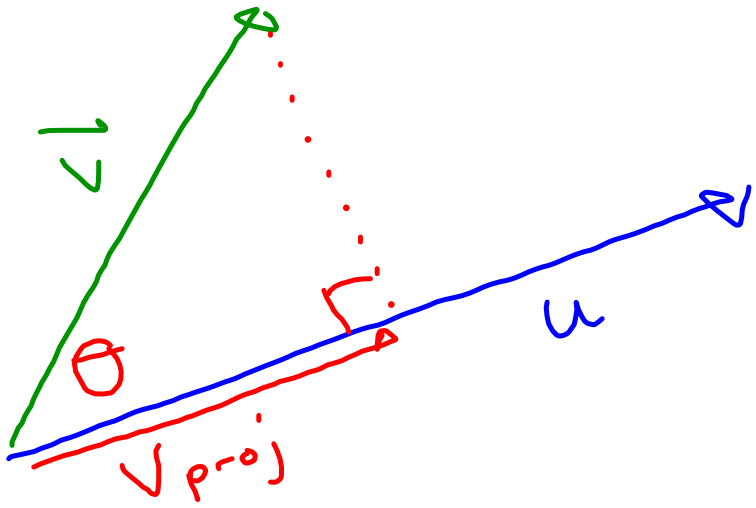
$$= \frac{5}{5\sqrt{5}}$$

$$\cos\theta = \frac{1}{\sqrt{5}}$$

$$\boxed{\theta = 63.4^\circ}$$

② Vector Projections

The projection of $\vec{v}$ onto $\vec{u}$ is the "shadow" that $\vec{v}$ casts onto $\vec{u}$.



If $0° \leq \theta < 90°$, the $\text{proj}_{\vec{u}} \vec{v}$ is in the direction of $\vec{u}$.

If $90° < \theta \leq 180°$, the $\text{proj}_{\vec{u}} \vec{v}$ is in the opposite direction of $\vec{u}$.

If $\theta = 90°$, there is no shadow, so $\text{proj}_{\vec{u}} \vec{v} = 0$

$$\text{proj}_{\vec{u}} \vec{v} = \left(\frac{\vec{v} \cdot \vec{u}}{\vec{u} \cdot \vec{u}} \right) \vec{u}$$

Ex. 2: Find the projection of $\vec{v}$ onto $\vec{u}$ if

$|\vec{u}| = 8, \quad |\vec{v}| = 12, \quad \theta = 39^\circ$

$$\text{proj}_{\vec{u}}\vec{v} = \left(\frac{\vec{v}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\right)\vec{u}$$

$$= \frac{|\vec{v}||\vec{u}|\cos\theta}{|\vec{u}|^2}\,\vec{u}$$

$$= \frac{|\vec{v}|\cos\theta}{|\vec{u}|}\,\vec{u}$$

$$= \frac{12\cos 39}{8}\,\vec{u}$$

$$= \frac{9.32}{8}|8\vec{u} = \boxed{9.32}$$

Ex. 3: Find the projection of $\vec{a} = [3, -5]$ onto $\vec{b} = [-1, 8]$

$$\text{proj}_{\vec{b}}\vec{a} = \left(\frac{\vec{a} \cdot \vec{b}}{\vec{b} \cdot \vec{b}}\right)\vec{b}$$

$$= \left[\frac{(3)(-1) + (-5)(8)}{(-1)(-1) + (8)(8)}\right][-1, 8]$$

$$= \left(-\frac{43}{65}\right)[-1, 8]$$

$$= \left[\frac{43}{65}, \frac{-344}{65}\right]$$

③ Work

Work = force × displacement

Ex. 4 : An object is dragged 5m up a ramp under a force of 30N applied at an angle of 40° to the ramp. Find the work done.

$$W = |f||d|\cos\theta$$

$$= 30(5)\cos 40^\circ$$

$$\boxed{W = 114.9\,J}$$

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5, 9, 11, 14, 15, 18, 19, 20