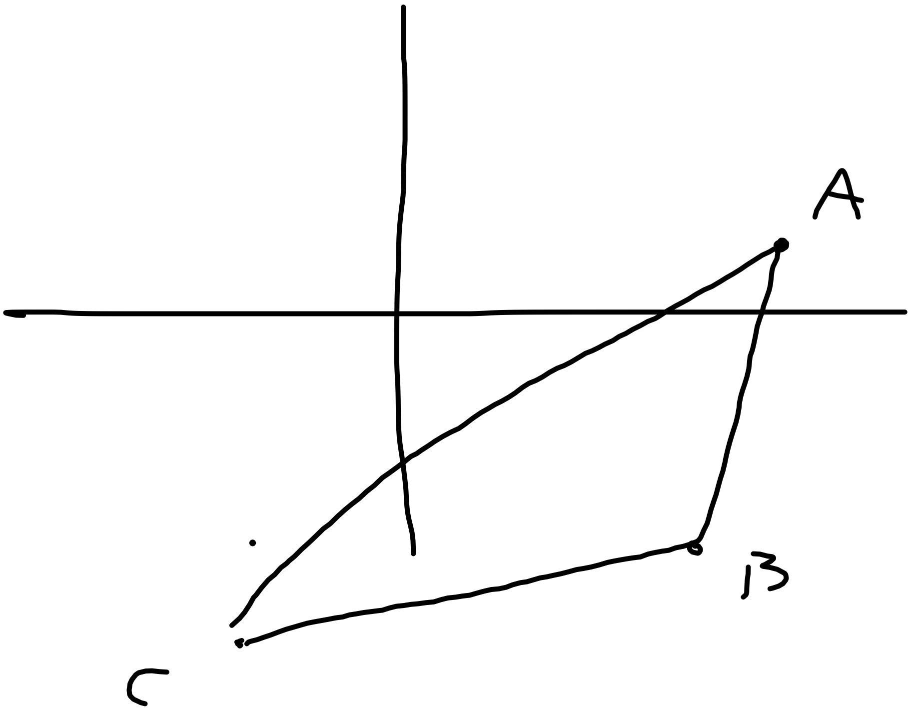
9) Interior angles of the

A(5, 1), B(4, −7), C(−1, −8)



make point $C \to (0,0)$

$\therefore$ add 1 to x, 8 to y

<u>or</u>

$$\overline{AC} = (5-(-1),\ 1-(-8))$$

$$= (6,\ 9)$$

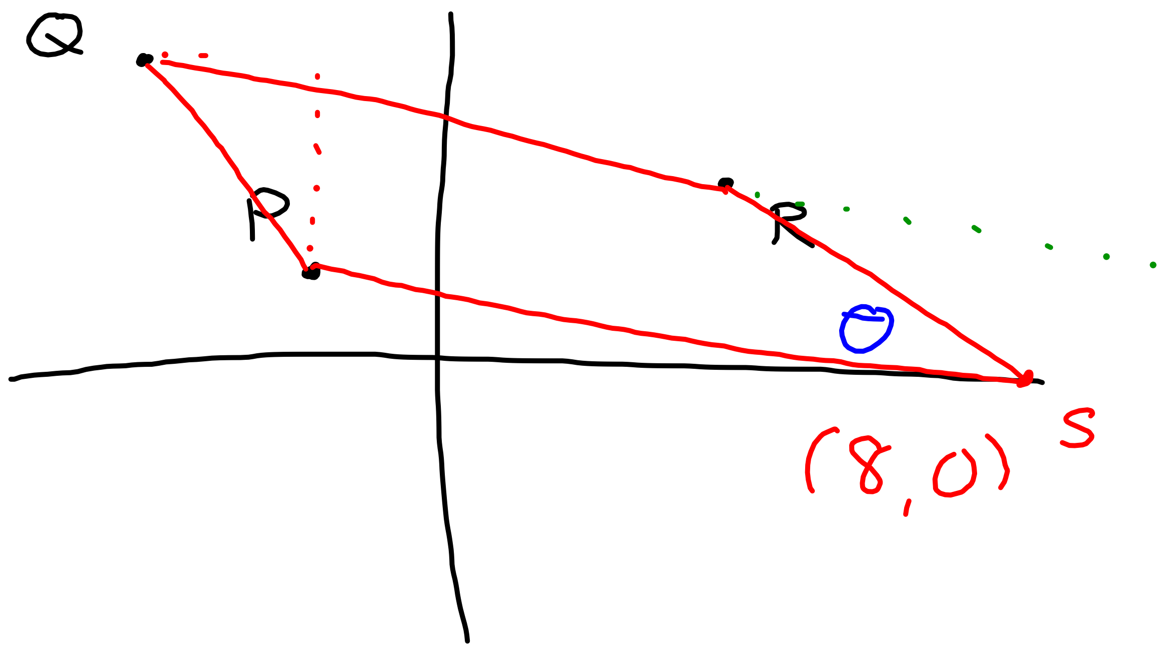
11

P(-2, 1)

Q(-6, 4)

R(4, 3)

a) Find S



For $\angle S$,

$\overline{SR}$ and $\overline{SP}$

$[4-8, 3-0]$

$\boxed{R - S}$

$SR = [-4, 3]$

$= [-2-8, 1-0]$

$= [-10, 1]$

$$\cos\theta = \frac{\overline{SR} \circ \overline{SP}}{|\overline{SR}||\overline{SP}|}$$

$$= \frac{(-4)(-10)+(3)(1)}{\sqrt{(-4)^2+3^2} \cdot \sqrt{(-10)^2+1^2}}$$

$$= \frac{40+3}{\sqrt{25} \cdot \sqrt{101}}$$

$\boxed{\theta = 31.2^\circ}$

For $\angle Q$, we need

$\overline{QP}$ and $\overline{QR}$

$= (-6-(-2), 4-1)$

$= [+4, -3]$

$(4-(-6), 3-4)$

$= [10, -1]$

$$\cos\theta = \frac{\overline{PQ} \circ \overline{QR}}{|\overline{PQ}||\overline{QR}|}$$

$$= \frac{(+4)(10)+3(-1)}{\sqrt{(-4)^2+3^2}\sqrt{10^2+(-1)^2}}$$

$= 148.84$

Three Dimensional Vectors

If $[a, b, c]$ is a 3D-vector,

$$\vec{i} = [1, 0, 0]$$
$$\vec{j} = [0, 1, 0]$$
$$\vec{k} = [0, 0, 1]$$

These are the unit vectors, and $[a, b, c] = a\vec{i} + b\vec{j} + c\vec{k}$

Magnitude of A Cartesian Vector

$$\vec{u} = [a, b, c],$$

$$|\vec{u}| = \sqrt{a^2 + b^2 + c^2}$$

Vector Addition

$$\vec{u} = [u_x, u_y, u_z]$$

$$\vec{w} = [w_x, w_y, w_z]$$

$$\vec{u} + \vec{w} = [u_x + w_x, u_y + w_y, u_z + w_z]$$

Vector Between Two Points

$\vec{u} = [u_x, u_y, u_z]$

$\vec{w} = [w_x, w_y, w_z]$

$\overrightarrow{uw} = [w_x - u_x, w_y - u_y, w_z - u_z]$

Ex. 1: Find the magnitude of $-3\vec{u} + \vec{v} - 5\vec{w}$ if

$\vec{u} = [3, 4, 5]$

$\vec{v} = [-2, 1, 6]$

$\vec{w} = [1, 4, -3]$

$$-3\vec{u} = [-9, -12, -15]$$
$$\vec{v} = [-2, 1, 6]$$
$$-5\vec{w} = [-5, -20, 15]$$

$$\vec{r} = -3\vec{u} + \vec{v} - 5\vec{w}$$
$$= [-16, -31, 6]$$

$$|\vec{r}| = \sqrt{(-16)^2 + (-31)^2 + (6)^2}$$
$$= \sqrt{256 + 961 + 36}$$

$$\boxed{|\vec{r}| = \sqrt{1253}}$$

Dot Product In 3D

If $\vec{a} = [a_x, a_y, a_z]$

$\vec{b} = [b_x, b_y, b_z]$

$$\boxed{\vec{a} \circ \vec{b} = a_x b_x + a_y b_y + a_z b_z}$$

Two vectors are orthogonal if $\vec{a} \circ \vec{b} = 0$.

Ex. 2: Find the angle between $P[-2, 3, 5]$ and $Q[4, -1, 4]$.

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{-2(4) + 3(-1) + 5(4)}{\sqrt{(-2)^2 + 3^2 + 5^2}\sqrt{4^2 + (-1)^2 + 4^2}}$$

$$= \frac{9}{\sqrt{38}\sqrt{33}} = \frac{9}{\sqrt{1254}}$$

$$\boxed{\theta = 75.3^\circ}$$

Ex.3: Find p and q such that $\vec{u} = [p, 3, 4]$ and $\vec{v} = [3, 3, q]$ are orthogonal.

$$\vec{u} \circ \vec{v} = 0$$

$$3p + 9 + 4q = 0$$

$$3p + 4q = -9$$

$$p = -3, \quad q = 0$$

$$p = 0, \quad q = -\frac{9}{4}$$

$$\boxed{p = 1}$$

$$3(1) + 4q = -9$$

$$4q = -12$$

$$\boxed{q = -3}$$

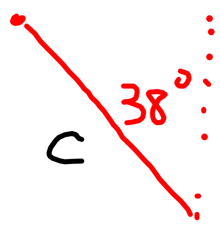
The key to solving 3D word problems are to either:

a) break into components

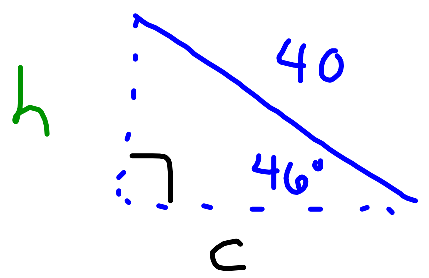
b) break into 2D triangles

Ex. 4: Batman shoots a line [N 38° W] at an angle of elevation of 46° that travels 40m. Break this into components.

Top view



Side view



Let c = connected side

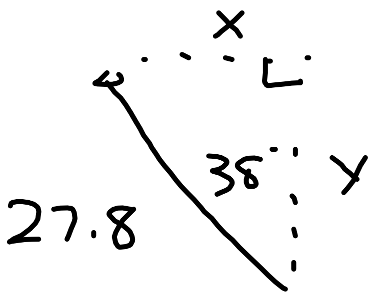
h = height

$$\sin 46^\circ = \frac{h}{40}$$

$$\boxed{h = 28.8\,\text{m [up]}}$$

$$\cos 46^\circ = \frac{c}{40}$$

$$\boxed{c = 27.8\,\text{m}}$$



$$\sin 38^\circ = \frac{x}{27.8} \qquad \cos 38^\circ = \frac{y}{27.8}$$

$$\boxed{x = 17.1\,\text{m [W]}} \qquad \boxed{y = 21.9\,\text{m [N]}}$$

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#4, 5, 6, 9, 11, 26, 35, 36