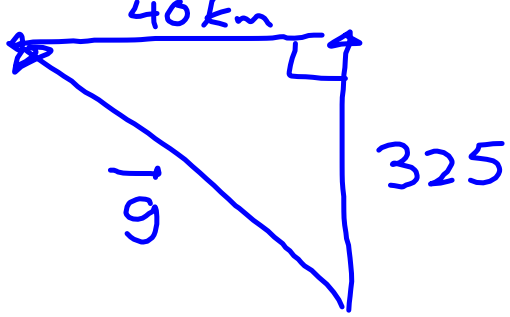
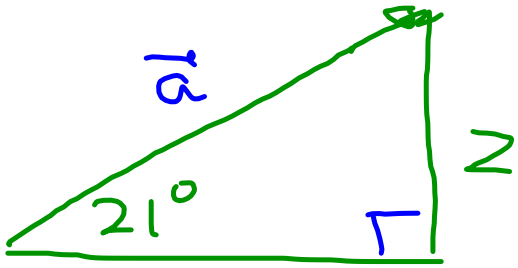
March 23, 2017

Warmup: Superman flies 325km/h [N] then is blown 40km/h [W]. If his angle of elevation is 21°, find the magnitude of his air and ground velocity.

top (birds eye)



side view



Let $\vec{g}$ = ground velocity

$\vec{a}$ = air velocity

$[-40, 325, 0]$ $[-40, 325, z]$

$$|\vec{g}| = \sqrt{40^2 + 325^2}$$

$$|\vec{g}| = \sqrt{107\,225}$$

$$\boxed{|\vec{g}| = 327.45}$$

$$\cos 21^\circ = \frac{327.45}{|\vec{a}|}$$

$$|\vec{a}| = \frac{327.45}{\cos 21}$$

$$\boxed{|\vec{a}| = 350.7}$$

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26b) Find the angle between our new vector and the 35N vector.

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$$

$|\vec{u}| = 58.7$

$|\vec{v}| = 35$

$\vec{u} = [25, 35, 40]$

$\vec{v} = [0, 35, 0]$

$$\cos\theta = \frac{25(0) + 35(35) + 40(0)}{(58.7)(35)}$$

$$= \frac{35^2}{35(58.7)}$$

$$\boxed{\theta = 53.4^\circ}$$

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The Cross Product

For vectors $\vec{u}$ and $\vec{v}$, we define the cross product of $\vec{u}$ and $\vec{v}$ as

$$\vec{u} \times \vec{v} = (|\vec{u}||\vec{v}|\sin\theta)\hat{n},$$

θ is the angle between $\vec{u}$ and $\vec{v}$

$\hat{n}$ is a unit vector perpendicular to $\vec{u}$ and $\vec{v}$

Ex. 1 : If $|\vec{u}| = 8$, $|\vec{v}| = 5$, and $\theta = 30°$, find $\vec{u} \times \vec{v}$

$$\vec{u} \times \vec{v} = (|\vec{u}||\vec{v}| \sin\theta)\hat{n}$$

$$= 8 \cdot 5 \cdot \sin 30° \hat{n}$$

$$= 20\hat{n}$$

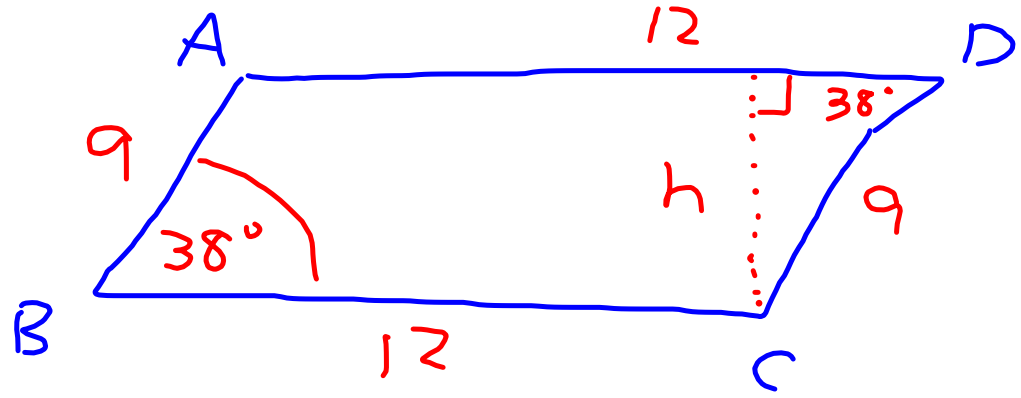
General Rules

$$\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$$

$$k(\vec{a} \times \vec{b}) = k\vec{a} \times \vec{b} = \vec{a} \times (k\vec{b})$$

Ex. 2 : If a parallelogram ✱ is formed by $\theta = 38°$, $|\overline{AB}| = 9$, $|\overline{BC}| = 12$, $|\overline{CD}| = 9$, $|\overline{DA}| = 12$, find the area of the parallelogram.



$$\sin 38 = \frac{h}{9}$$

$$h = 9 \cdot \sin 38^\circ$$

Area of a Parallelogram

$$= 12 \cdot 9 \cdot \sin 38^\circ$$

$$= \overline{AB} \times \overline{BC} \quad (*)$$

$$\text{Area} = 66.5$$

Cartesian Cross Product

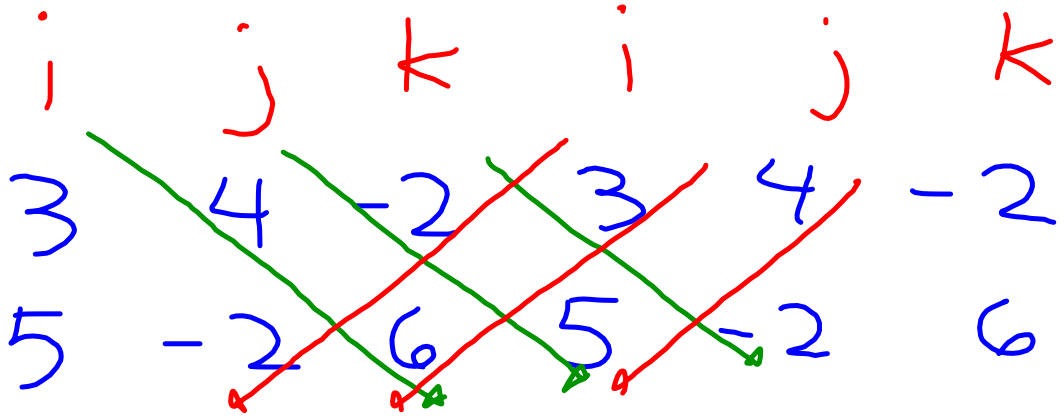
Let $\vec{a} = [a_x, a_y, a_z]$

$\vec{b} = [b_x, b_y, b_z]$

$$\vec{a} \times \vec{b} = [a_y b_z - a_z b_y, \; a_z b_x - a_x b_z, \; a_x b_y - a_y b_x]$$

Ex. 3: If $\vec{a} = [3, 4, -2]$, $\vec{b} = [5, -2, 6]$.

find $a \times b$



$4(6)i + (-2)(5)j + 3(-2)k$

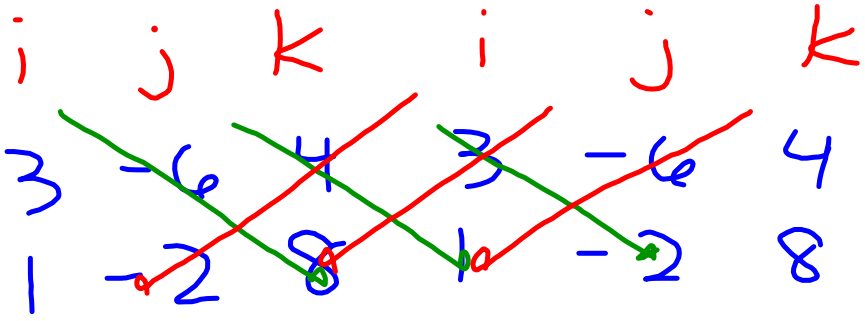
$-(-2)(-2)i - 3(6)j - 4(5)k$

$= 24i - 4i - 10j - 18j - 6k - 20k$

$= 20i - 28j - 26k$

$= [20, -28, -26]$

Ex. 4: If $\vec{u} = [3, -6, 4]$,
$\vec{v} = [1, -2, 8]$,
find $\vec{u} \times \vec{v}$



$8(-6)i + 4(1)j + 3(-2)k$
$-(4)(-2)i - 3(8)j - 1(-6)k$

$= -48i + 8i + 4j - 24j - 6k + 6k$

$= -40i - 20j + 0k$

$= [-40, -20, 0]$

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#7, 8, 9, 11, 12, 16

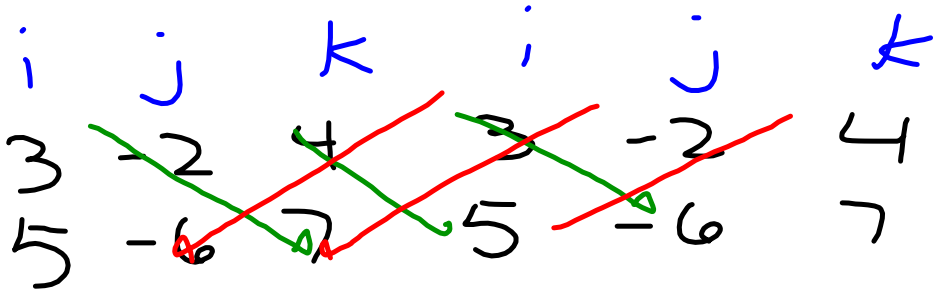
Ex. 5: If $\vec{a} = [3, -2, 4]$ and $\vec{b} = [5, -6, 7]$ find θ using the cross product.

$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \, \hat{n}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$$

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$$

$a \times b$



$7(-2)i + 4(5)j + 3(-6)k$

$-4(-6)i - 3(7)j - (-2)(5)k$

$= -14i + 24i + 20j - 21j - 18k + 10k$

$= [10, -1, -8]$

$|\vec{a} \times \vec{b}| = \sqrt{10^2 + (-1)^2 + (-8)^2} = \sqrt{165}$

$|\vec{a}| = \sqrt{3^2 + (-2)^2 + 4^2} = \sqrt{29}$

$|\vec{b}| = \sqrt{5^2 + (-6)^2 + 7^2} = \sqrt{110}$

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$$

$$= \frac{\sqrt{165}}{\sqrt{29}\sqrt{110}}$$

$$\boxed{\theta = 13.1^\circ}$$