March 24, 2017

Warmup: If $\vec{u} = [2, 3, -4]$, $\vec{v} = [5, 6, -7]$ find

a) $\vec{u} \cdot \vec{v}$

$= 2(5) + 3(6) + (-4)(-7)$

$= 10 + 18 + 28$

$= \boxed{56}$

b) $\vec{u} \times \vec{v}$

$$\begin{array}{cccccc} i & j & k & i & j & k \\ 2 & 3 & -4 & 2 & 3 & -4 \\ 5 & 6 & -7 & 5 & 6 & -7 \end{array}$$

$$i(3)(-7) + (-4)(5)j + 2(6)k$$

$$-(-4)(6)i - (2)(-7)j - 3(5)k$$

$$= -21i + 24i - 20j + 14j + 12k - 15k$$

$$= 3i - 6j - 3k$$

$$= [3, -6, -3]$$

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16b) Prove

$$a \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$\vec{u} \times \vec{v} = |\vec{u}||\vec{v}| \sin\theta \, \hat{n}$$

$$a \times [b_x + c_x, b_y + c_y, b_z + c_z]$$

i	j	k	i	j	k
a_x	a_y	a_z	a_x	a_y	a_z
$b_x + c_x$	$b_y + c_y$	$b_z + c_z$			

Applications of the Dot and Cross Product

Torque

Torque is the amount of force upwards or downwards applied by rotation

$$|\vec{\tau}| = |\vec{r}||\vec{F}|\sin\theta$$

$\vec{r}$ = radius

$\vec{F}$ = force

$$|\vec{\tau}| = |\vec{r} \times \vec{F}|$$

The direction of the orthogonal force is like opening a bottle:

upward force: counterclockwise

downward force: clockwise

To increase the torque, you can either

a) increase the force

b) increasing the radius (distance) of the force applied

Ex. 1: A force of 80N is applied at 70° to the handle 25 cm from the centre of a bolt clockwise. What is the magnitude of the torque and in what direction?

$$|\tau| = |\vec{r}||\vec{F}|\sin\theta$$

$$= 0.25 \cdot 80 \cdot \sin 70°$$

$$\boxed{|\tau| = 18.8}$$

direction downwards

Projections

Projections in 3D are the same as in 2D.

Ex. 2: If $\vec{u} = [2, -6, 8]$, $\vec{v} = [-3, 4, 5]$ find the projection of $\vec{v}$ onto $\vec{u}$.

$$\text{proj}_{\vec{u}}\vec{v} = \left(\frac{v \cdot u}{u \cdot u}\right)\vec{u}$$

$$= \frac{(-3)(2)+(-6)(4)+8(5)}{(2)^2+(-6)^2+8^2}[2,-6,8]$$

$$= \frac{10}{104}[2,-6,8]$$

$$= \frac{5}{52}[2,-6,8]$$

$$= \left[\frac{10}{52}, -\frac{30}{52}, \frac{40}{52}\right]$$

$$= \left[\frac{5}{26}, \frac{-15}{26}, \frac{10}{13}\right]$$

Triple Scalar Product

* This has several applications, including the volume of a parallelepiped (a 3D parallelogram. The order of operations are to cross before you dot so the result is a scalar

$$\vec{a} \circ \underbrace{\vec{b} \times \vec{c}}_{1st}$$

Ex. 3: If $\vec{u} = [5, -2, 3]$, $\vec{v} = [-1, 6, 4]$, $\vec{w} = [3, 4, -2]$, find $\vec{u} \cdot \vec{v} \times \vec{w}$.

$\vec{v} \times \vec{w}$

$$\begin{array}{cccccc} i & j & k & i & j & k \\ -1 & 6 & 4 & -1 & 6 & 4 \\ 3 & 4 & -2 & 3 & 4 & -2 \end{array}$$

$$6(-2)i + 4(3)j + (-1)(4)k$$

$$-4(4)i - (-1)(-2)j - 6(3)k$$

$$-12i - 16i + 12j - 2j - 4k - 18k$$

$$= [-28, 10, -22]$$

$\vec{u} \circ (\vec{v} \times \vec{w})$

$= (5)(-28) + (-2)(10) + 3(-22)$

$= -140 - 20 - 66$

$= -226$

Area of A Triangle

$$A = \frac{1}{2} |\vec{a} \times \vec{b}|$$

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#4, 5, 6, 7, 8, 9, 17