

March 30, 2017

Vectors and Parametric
Equations of Lines in 2D

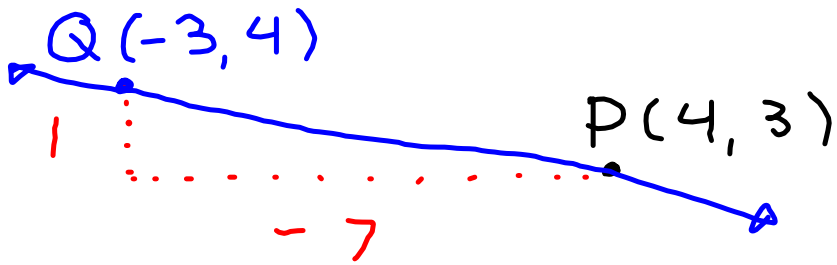
To find these, we must be given either

a) two points

b) one point and a vector that defines the direction of a line (slope)

A **direction vector** is a nonzero vector $\vec{m} = [a, b]$ parallel to the given line.

Ex. 1: A line passing through $P(4, 3)$ has $\vec{m} = [-7, 1]$ as its direction vector. Sketch this line.



Ex. 2 : A line passes through

$A(\frac{1}{2}, -3)$ and $B(\frac{3}{4}, \frac{1}{2})$.

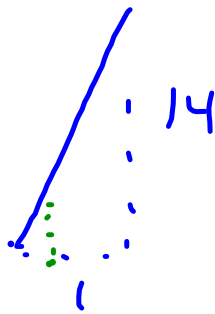
Determine a direction vector for this line.

$$\vec{m} = \overrightarrow{AB} = [b_x - a_x, b_y - a_y]$$
$$= [3/4 - 1/2, 1/2 - (-3)]$$

$$\vec{m} = [1/4, 3 1/2]$$

x4
you can multiply x and y
to get integers for slope

$$\vec{m} = [1, 14]$$



Expressing Equations of Lines Using Vectors

Vector equation of A Line

$$\vec{r} = \vec{r}_0 + \lambda \vec{m}$$

line = point + slope

$$m = [a, b], r_0 = [x_0, y_0]$$

$$r = [x, y], \lambda \in \mathbb{R}$$

The parametric equation of
a line is

$$x = x_0 + \lambda a$$

$$y = y_0 + \lambda b$$

Ex. 3: Determine the vector and parametric equations of a line passing through $A(1, 4)$ with direction vector $\vec{m} = [-3, 3]$

a) Sketch the line with four points

b) Is $Q(-21, 23)$ or

$R(-29, 34)$ on this line?

Solution

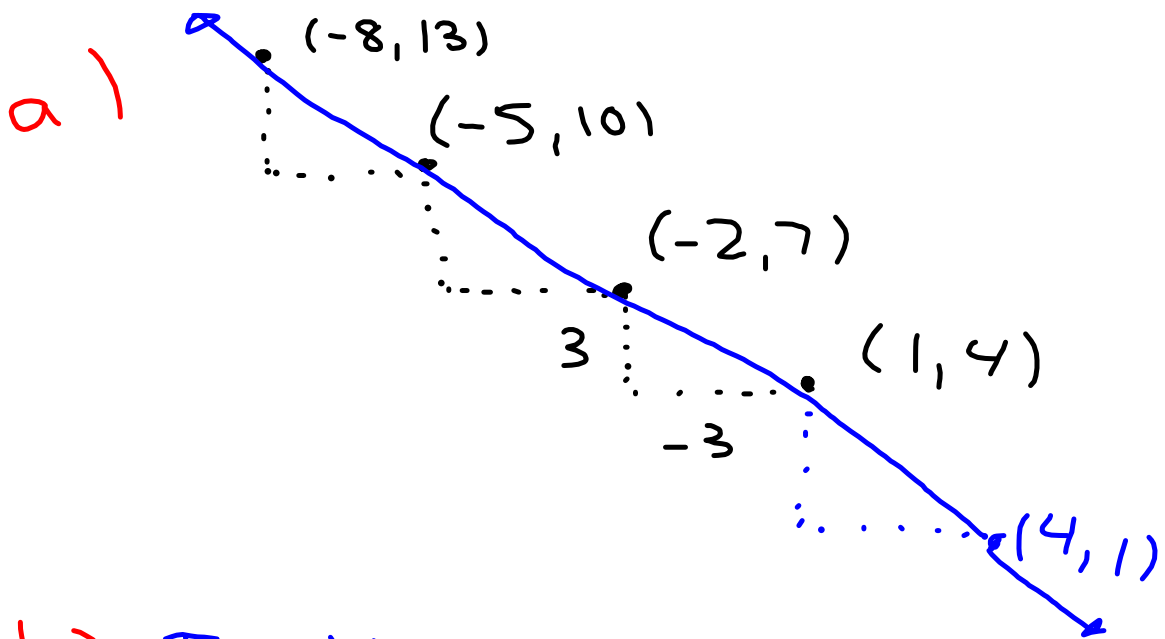
Vector equation of the line

$$\vec{r} = (1, 4) + t[-3, 3]$$

Parametric equation

$$x = 1 - 3t$$

$$y = 4 + 3t$$



b) Is the point $(-21, 23)$ on the line?

sub $(-21, 23)$ into your parametric equation and solve for t . If the t 's match, it is on the line.

$$-21 = 1 - 3t \quad ; \quad 23 = 4 + 3t$$

$$\frac{-22}{-3} = \frac{-3t}{-3} \quad ; \quad \frac{19}{3} = \frac{3t}{3}$$

$$\boxed{\frac{22}{3} = t}$$

$$\boxed{t = \frac{19}{3}}$$

\therefore not on same line.

Is the point $(-29, 34)$ on the line?

$$-29 = 1 - 3t \quad ; \quad 34 = 4 + 3t$$

$$\frac{-30}{-3} = \frac{-3t}{-3} \quad ; \quad 30 = 3t$$

$$\boxed{t = 10}$$

$$\boxed{t = 10}$$

Ex.4: Determine the vector and parametric equations of the line containing the points $E(-1, 5)$ and $F(6, 11)$.

$$\vec{m} = \vec{EF} = [6 - (-1), 11 - 5]$$

$$\boxed{\vec{m} = [7, 6]}$$

Vector Equation

$$\vec{r} = (-1, 5) + t[7, 6]$$

Parametric

$$x = -1 + 7t$$

$$y = 5 + 6t$$

b) What are the coordinates when this crosses the x-axis?

when $y = 0$

Use your parametric!

$$0 = 5 + 6t$$

$$\boxed{t = -\frac{5}{6}}$$

$$\text{sub } t = -\frac{5}{6} \text{ in}$$

$$x = -1 + 7\left(-\frac{5}{6}\right)$$

$$= -1 - \frac{35}{6}$$

$$\boxed{x = -\frac{41}{6}}$$

\therefore it intersects at the
point $\left(-\frac{41}{6}, 0\right)$.

Vectors in 3 Space

The vector equation of a line

$$\vec{r} = \vec{r}_0 + \lambda \vec{m}, \lambda \in \mathbb{R}$$

$$\vec{r}_0 = [x_0, y_0, z_0]$$

$$\vec{r} = [x, y, z]$$

$$\vec{m} = [a, b, c]$$

The parametric equation of a line is:

$$x = x_0 + ta$$

$$y = y_0 + tb$$

$$z = z_0 + tc$$

Ex.5: A line passes through

⊗ $A(-2, 3, 4)$ and $B(1, 5, -1)$

a) Write a vector equation of the line

$$\vec{m} = \vec{AB} = [1 - (-2), 5 - 3, -1 - 4]$$

$$\vec{m} = [3, 2, -5]$$

Vector Equation

$$\vec{r} = (1, 5, -1) + t[3, 2, -5]$$

b) Write the parametric equation of the line

$$x = 1 + 3t$$

$$y = 5 + 2t$$

$$z = -1 - 5t$$

c) Will $(6, 3, -2)$ lie on the line?

$$\begin{array}{l}
 6 = 1 + 3x \quad ; \quad 3 = 5 + 2x \quad ; \quad -2 = -1 - 5x \\
 5 = 3x \quad ; \quad -2 = 2x \quad ; \quad -1 = -5x \\
 \boxed{x = \frac{5}{3}} \quad ; \quad \boxed{x = -1} \quad ; \quad \boxed{x = \frac{1}{5}}
 \end{array}$$

\therefore not on the same line.

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7, 11, 12, 20, 23, 24