March 31, 2017

11d) $8x + 9y = -45$

choose 2 points

$8x + 9(0) = -45$

$$x = -\frac{45}{8}$$

$A\left(-\frac{45}{8}, 0\right)$

$$8(0) + 9y = -45$$

$$\boxed{y = -5}$$

$$\boxed{B(0, -5)}$$

$$\vec{m} = \overrightarrow{AB} = \left[0 - \left(-\frac{45}{8}\right), -5 - 0\right]$$

$$= \left[\frac{45}{8}, -5\right]$$

$$= [45, -40]$$

$$\boxed{\vec{m} = [9, -8]}$$

Vector equation

$$\vec{r} = (0, -5) + t[9, -8]$$

Parametric

$$x = 9t$$

$$y = -5 - 8t$$

More points

choose $t = 1, 2, 3, \ldots$

24a)

$$[x, y, z] = [3, -1, 5] + t[2, -3, -1]$$

$$\vec{a} \cdot \vec{b} = 0$$

$$[2, -3, -1] \cdot [x, y, z] = 0$$

$$[2, -3, -1] \cdot [3, 2, 0]$$

Vector and Parametric Equations of the Plane

Planes are flat surfaces that extend infinitely far in all directions.

To represent planes, parallelograms are used to represent small parts of the plane, designated as Π.

A plane can be determined given any of the following:

1. a line and a point not on the line
2. three non collinear points
3. two intersecting lines
4. two parallel and non-coincident lines

Vector Equation of A Plane

$$\Pi = (x_0, y_0, z_0) + s[a_1, a_2, a_3] + t[b_1, b_2, b_3]$$

(point plus two direction vectors)

Parametric Equation

$$x = x_0 + sa_1 + tb_1$$

$$y = y_0 + sa_2 + tb_2$$

$$z = z_0 + sa_3 + tb_3$$

Ex. 1: Determine the vector and parametric equations of the plane for points

$A(-1, 3, 8)$

$B(-1, -1, 0)$

$C(4, 1, 1)$

$$\overrightarrow{AB} = [-1-(-1), -1-3, 0-8]$$

$$= [0, -4, -8]$$

$$\boxed{\overrightarrow{AB} = [0, 1, 2]}$$

$\overline{BC} = [4-(-1), 1-(-1), 1-0]$

$\overline{BC} = [5, 2, 1]$

Vector

$\Pi : (4,1,1) + s[0,1,2] + t[5,2,1]$

Parametric

$x = 4 + 5t$

$y = 1 + s + 2t$

$z = 1 + 2s + t$

Ex. 2 : Determine the vector and parametric equations of the plane containing the point $P(1,-5,9)$ and

$L: \vec{r} = (1,1,1) + s[-1,1,0]$

$\overrightarrow{PA} = [1-1, 1-(-5), 1-9]$

$= [0, 6, -8]$

$\boxed{\overrightarrow{PA} = [0, 3, -4]}$

Vector Equation of the Plane

$$\pi : (1,1,1) + s[-1,1,0] + t[0,3,-4]$$

Parametric Equation

$$x = 1 - s$$

$$y = 1 + s + 3t$$

$$z = 1 - 4t$$

Ex.3: The plane

$\Pi: (6,-2,-3) + s[1,3,0] + t[2,2,-1]$

Find the poi between Π and the z-axis.

Looking for the point when $x = 0$, $y = 0$.

$x = 6 + s + 2t$

$y = -2 + 3s + 2t$

$z = -3 - t$

$0 = 6 + s + 2t$

$0 = -2 + 3s + 2t \quad \times -1$

$0 = 8 - 2s$

$2s = 8$

$\boxed{s = 4}$

$0 = 6 + 4 + 2t$

$-10 = 2t$

$\boxed{t = -5}$

To get z, sub $t = -5$

$z = -3 - (-5)$

$\boxed{z = 2}$

$\therefore$ POI is $(0, 0, 2)$.

HW: p. 451

6, 7, 10, 15, 17