Sept. 8, 2017

Warmup: Find the vertex of

$y = 3x^2 - 9x + 1$

and state domain and range.

$y = 3(x^2 - 3x) + 1$

$\hookrightarrow \left(-\frac{3}{2}\right)^2 = \boxed{\frac{9}{4}}$

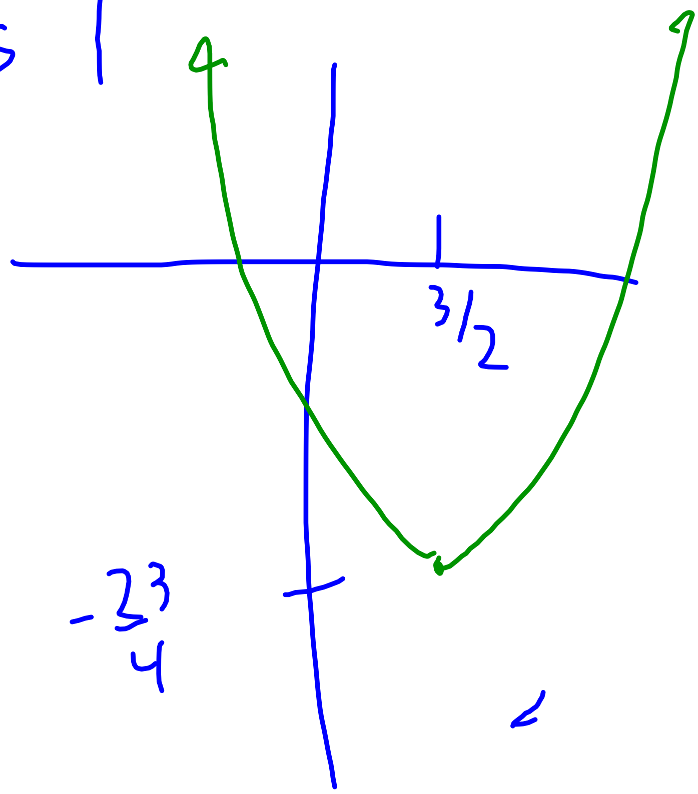
$$y = 3\left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right) + 1$$

$$y = 3\underline{\left(x^2 - 3x + \frac{9}{4}\right)} - \frac{27}{4} + \frac{4}{4}$$

$$y = 3\left(x - \tfrac{3}{2}\right)^2 - \frac{23}{4}$$

vertex: $\left(\frac{3}{2}, -\frac{23}{4}\right)$

opens ↑



$d: \{x \mid x \in \mathbb{R}\}$

$r: \{y \mid y \geq -\frac{23}{4}, y \in \mathbb{R}\}$

Fast Way To Find Vertex

$$y = ax^2 + bx + c$$

$$x = -\frac{b}{2a}$$

$$y = f\left(\frac{-b}{2a}\right)$$

$$y = 3x^2 - 9x + 1$$

up/down

$$x = \frac{-b}{2a} = \frac{-(-9)}{2(3)} = \frac{+9}{6} = \boxed{\frac{3}{2}}$$

$$y = f\left(\frac{3}{2}\right) = 3\left(\frac{3}{2}\right)^2 - 9\left(\frac{3}{2}\right) + 1$$

sub $3/2$ into equation

$$y = 3\left(\frac{9}{4}\right) - 9\left(\frac{3}{2}\right) + 1$$

$$= \frac{27}{4} - \frac{27}{2} + 1$$

$$= \frac{27}{4} - \frac{54}{4} + \frac{4}{4}$$

$$= \boxed{-\frac{23}{4}} \quad V: \left(\frac{3}{2}, -\frac{23}{4}\right)$$

Transformations

$$y = af(k(x-d)) + c$$

a = vertical stretch
(multiply y-values)

k = horizontal compression
(divide x-values by k)

d = horizontal shift
(subtract x by d)

c = vertical shift
(add c to y)

Ex. 1: Translate into English

$$y = -3f(2(x+3)) - 4$$

vertical stretch of -3
(multiply y by -3)
horizontal compression of 2
(divide x by 2)
horizontal shift of 3 left
(subtract 3 from x)
vertical shift 4 units down
(subtract 4 from y)

Polynomial Functions

A polynomial has the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots$$

Ex:

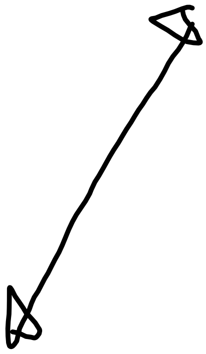
$$f(x) = 7x^3 + 5x^2 + 10x + 12$$

$$g(x) = 9x^5 - 8x^2 + x - 15$$

A power function of a polynomial is of the form $y = ax^n$, $n \in I$.

End Behaviour is what happens to the graph as it approaches infinity $(x \to \infty)$, or negative infinity $(x \to -\infty)$

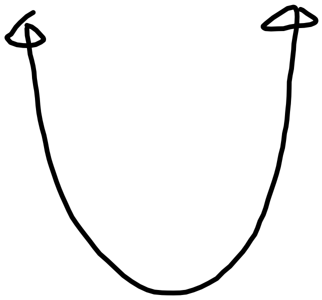
For odd functions
$(x, x^3, x^5, \dots)$ the
end behaviour is



(as $x \to \infty$, $y \to \infty$,
$x \to -\infty$, $y \to -\infty$)

For even functions
$(x^2, x^4, x^6, \dots)$ the end

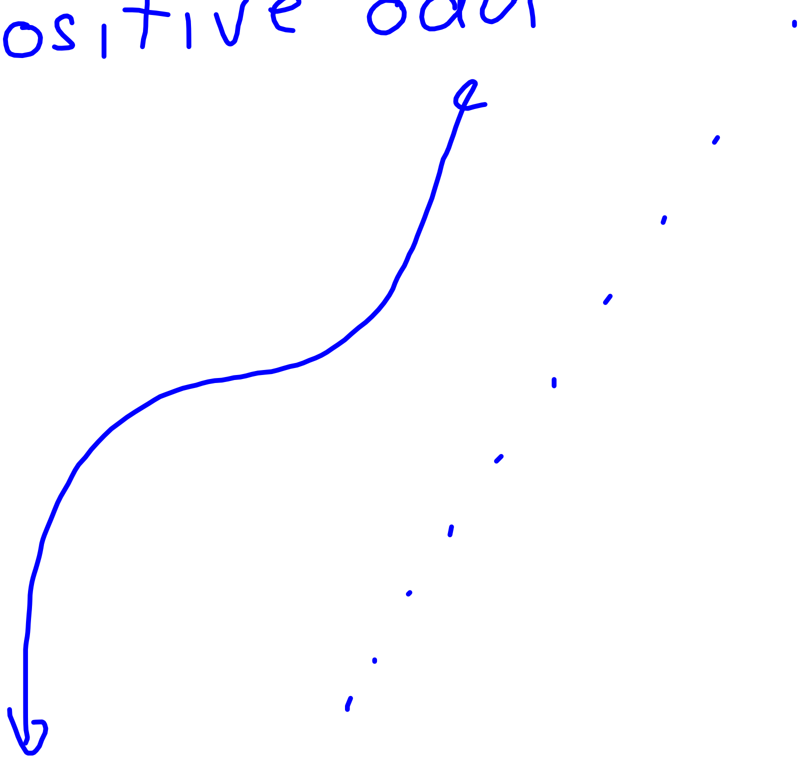
behaviour is



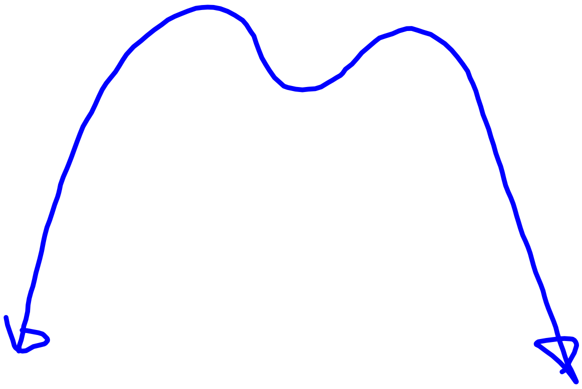
(as $x \to \infty$, $y \to \infty$,
$x \to -\infty$, $y \to \infty$)

Ex. 1: Do a rough sketch of the following

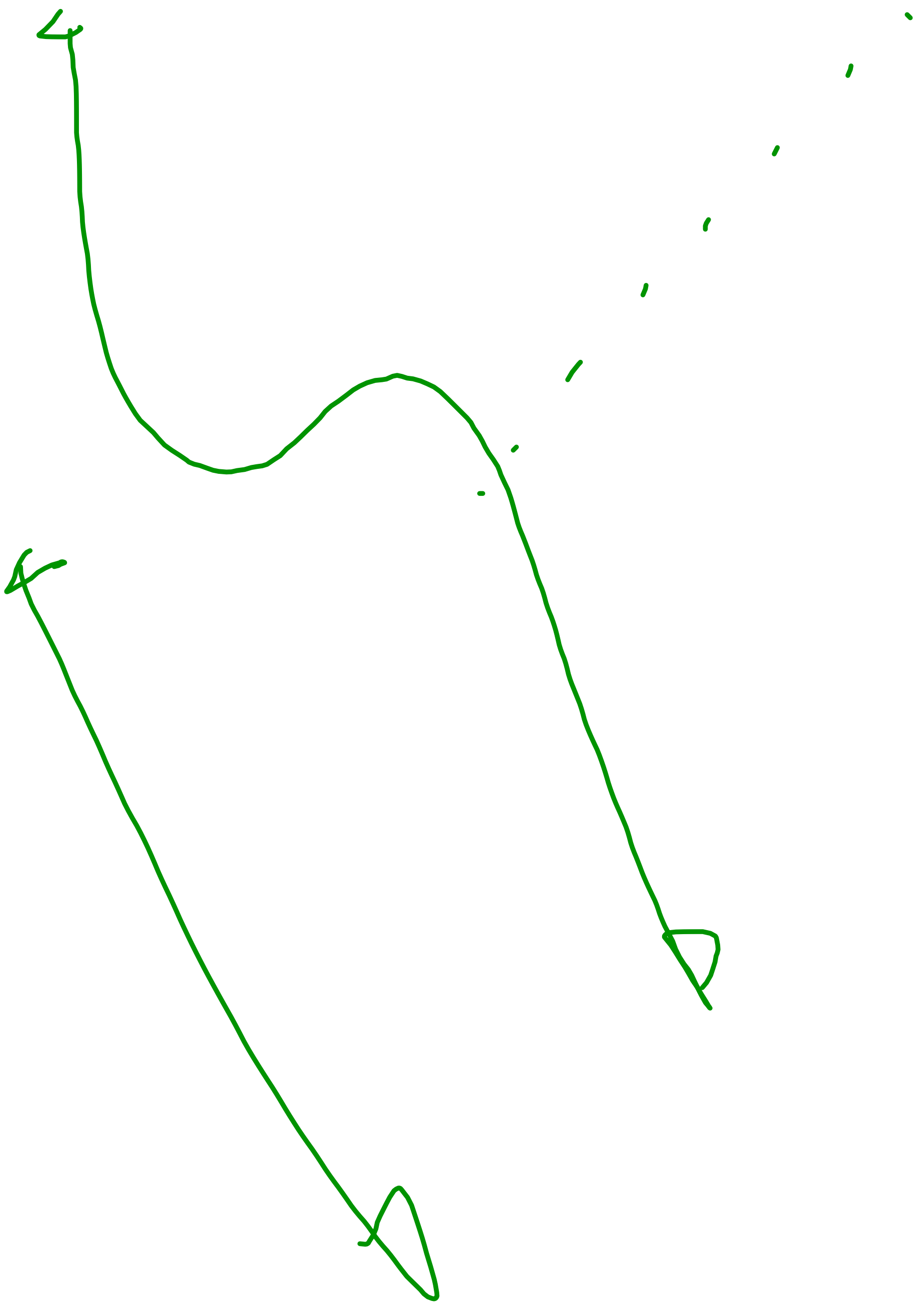
a) $y = \underbrace{2x^3}_{\text{positive odd}}$



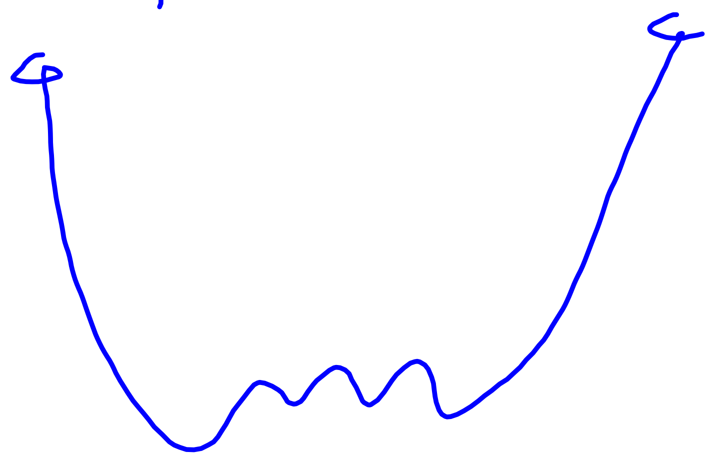
b) $y = \underbrace{-4x^{4}}_{\text{negative even}}$ even



c) $y = \underbrace{-2x^5}_{\text{neg odd function}} + 4x^4 + 3x^2$

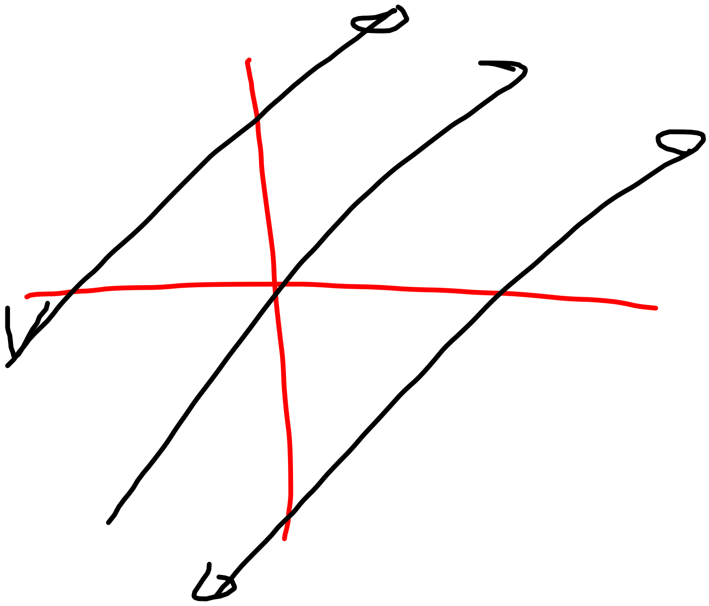


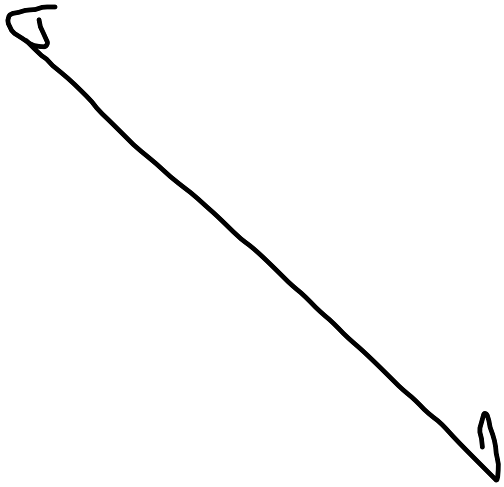
$$y = \underbrace{16x^{242}}_{\text{pos even}}$$



HW p. 12

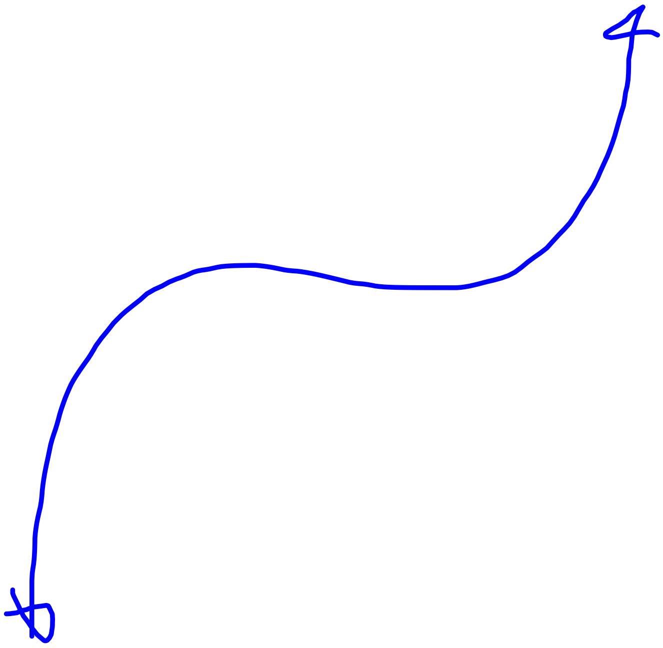
3, 4, 7, 16, 17





$$x \to -\infty, \; y \to -\infty$$

$$x \to \infty, \; y \to -\infty$$



$x \to -\infty, \; y \to -\infty$

$x \to \infty, \; y \to \infty$

$f(x)$ $-3f(x)$

$(0,1)$ $(0,-3)$

$(1,3)$ $(1,-9)$

$(2,9)$ $(2,-27)$

$y \neq 0$ $y \neq 0$